Things that Count

14 April 2014

Things that Count

The rise and fall of calculators

Jim Falk

things-that-count.net

Jim Falk:

Things that Count

The rise and fall of calculators

* * *

Typeset by Wikipublisher

http://metastudies.net/pmwiki/pmwiki.php?n=Site.PrintBook
Printed: 14 April 2014

Contents

Chapter 1

Introduction

1.1 Introduction

Figure 1.1. -

(Replica Pascaline "1652" - collection Calculant)

This short book outlines a 37,000 year story, one in which humans came to count, record their counting and do simple arithmetic sums. Over time these capabilities became essential ingredients in what became increasingly complex societies. More citizens needed to be able to manipulate numbers in their daily lives and, over time, a range of aides of various sorts were developed to assist. At the beginning, the aides were very simple - for example, marks scribed on bones and patterns arranged with pebbles. Later, primitive devices and tables were developed and sold. Over time, much more elaborate mechanical devices were developed to help in this task. Many of these devices, where they survive, now represent little more than mechanical fossils. Unused and largely forgotten their remains are scattered across history from earliest human pre-history to our present moment.

The need for calculation, however has prospered. As societies have become more complex, transactions in them depending on arithmetic (the familiar tasks of counting, adding, subtracting, multiplying and dividing), as well as more complex mathematics, have intensified. Yet over much of this period, for many people in these societies, doing even the simplest arithmetic tasks has been neither easy nor, for some, comprehensible. For this reason finding ways to do these tasks faster and more accurately, and to spread the ability across more people, has been a preoccupation in many societies. It is the approaches that have been taken to aid achieving these simple goals (rather than the development of complex mathematics) which is the primary focus of this website.

Early "calculators" were not things. Rather they were people who were employed to calculate. Over time these people were first aided, but later replaced by calculating devices. These devices became very widely used across many countries. There is evidence we may now be passing the heyday of such stand-alone calculators. This is because, increasingly since the advent of electronic computing, the aides to calculation have begun to appear in virtual form as apps in phones, tablets and laptops. The end of calculators, seen as devices, in this sense is looming.

One might imagine that a history of calculators would consist simply of the progressive discovery and invention of ever more effective and sophisticated calculating devices. Indeed many such accounts do focus on this with loving details of the minutae of mechanical invention. But to focus simply on that is to oversimplify and lose much of what is potentially interesting. Across human history many weird things were indeed devised for doing simple calculations. But the development of these begs a series of questions: When and why were they made, how were they used, and why at times were they forgotten for centuries or even millennia?

The objects described here, which are used to help illustrate answers to these sort of questions, are drawn from across some 4,000 years of history. Each of them was created with a belief that it could assist people in thinking about (and with) numbers. They range from little metal coin-like disks to the earliest electronic pocket calculators - representing a sort of 'vanishing point' for all that had come before. For convenience, I will refer to this set of artifacts as "collection Calculant" (which are described in more detail in an accompanying web site - "things-that-count.com").[1] The name of the collection is taken from the Latin meaning simply "they calculate"[2] The collection and the history it illustrates in a sense form a duet - the two voices each telling part of the story. The history has shaped what has been collected, and the collection has helped shape how the story is told.

1.1.1 Initial observations.

As most people know, the spread of electronic personal calculators of the 1970s was followed quickly by the first personal computers. Before long, computer chips began to be embodied in an ever expanding array of converging devices. In turn, ever greater computing power spread across the planet. However sophisticated these modern computers appeared on the outside, and whatever the diversity of functions they performed, at heart they achieved most of this by doing a few things extremely fast. (Central to the things they did were logic operations such as "if", "and", "or"

1. The full address is `http://things-that-count.com`. The author, Jim Falk, may be contacted through this website.
2. "Calculant" is the third-person plural present active of the Latin verb calculo (calculare, calculavi) meaning "they calculate, they compute".

and "not", and the arithmetic operations of addition and subtraction - from which multiplication and division can also be derived). On top of this were layers of sophisticated programming, memory and input and output.

Prior calculating technologies had to rely on slower mechanical processes. This meant they were much more limited in speed, flexibility and adaptability. Nevertheless they too were designed to facilitate the same fundamental arithmetic and logical operations. The technologies of mathematics are in this sense much simpler than the elaborate analytic structures which make up mathematical analysis. And for this reason, it is not necessary to consider all of mathematics in order to follow much of the history of how the technologies to aid mathematical reasoning developed. Just considering the history of the development of aids to calculation can tell a great deal. As already noted, it is that which is dealt with here.

The calculational devices that were developed show an unmistakeable progression in complexity, sophistication and style from the earliest to the latest. Corresponding to this it is possible to construct histories of calculational aids as some sort of evolution based on solving technical problems with consequent improvements in design building one upon the other. But, as already noted, it is also important to understand *why* they were invented and used.

Fortunately in order to understand what has shaped the development of these calculational aids we can largely avoid talking much about mathematics. This is lucky because mathematics is by now a huge field of knowledge. So you are entitled to relief that in this site we will avoid much of mathematics. We need not touch, for example, on calculus, set and group theory, the mathematics of infinite dimensional vector spaces that make the modern formulation of quantum mechanics possible, and tensors which Einstein used to express his wonderfully neat equations for the shape of space-time.[3] It will be sufficient to note that many modern challenges - from the prediction of climate under the stress of global warming, to the simulation of a nuclear reactor accident, to the deconstruction of DNA - could not occur without enormous numbers of calculations which in the end are constituted out of additions and subtractions (and multiplications and divisions) and can only be carried out in workable times with the use of ever faster calculating devices.

Even keeping our attention restricted to the basic arithmetic operations, it turns out we will still encounter some of the curly issues that we would have to think about if we were focussing on the whole evolving field of mathematical thought. Of course history of mathematics is itself a field of scholastic study which can be developed from many perspectives. These include those from the mainstream of history and philosophy of science[4] through to the sociology of science.[5] Even though this discussion here focuses on only a tiny "arithmetic core" of mathematics it will

3. See for example `http://mathworld.wolfram.com/EinsteinFieldEquations.html` or for more explanation `http://physics.gmu.edu/~joe/PHYS428/Topic9.pdf` (both viewed 26 Dec 2011)
4. See for example, Eleanor Robson, Jacqueline A. Stedall, *The Oxford handbook of the history of mathematics*, Oxford University Press, UK, 2009
5. See for example, Sal Restivo, *Mathematics in Society and History: Sociological Inquiries*, Kluwer Academic Publishers, Netherlands, 1992

still be useful to take some account of this literature and its insights. In particular, whether concerning ourselves with the evolution of the simple areas of mathematics or the more obstruse areas, one question is always raised: what led to this particular development happening as and when it did?

1.1.2 Did increases in the power of mathematics lead the development of calculators? Was it the other way round?

It might be assumed that arithmetic, and more broadly, mathematics, developed through a process that was entirely internal to itself. For example, this development might have been propelled forward because people could ask questions which arise within mathematics, but require the invention of new mathematics to answer them.

Suppose we know about addition and that 2+2 =4. Then it is possible to ask what number added to 4 would give 2. Answering that involves inventing the idea of a negative number. This leads to progress through ‘completing mathematics’ (i.e. seeking to answer all the questions that arise in mathematics which cannot yet be answered.) That must be part of the story of how mathematics develops. Yet the literature on the history of mathematics tells us this cannot be all.

The idea of ‘mathematics’, and doing it, are themselves inventions. The question of when mathematics might be useful will have different answers in different cultures. Different societies may identify different sorts of issues as interesting or important (and only some of these will be usefully tackled with mathematics). Also different groups of people in those societies will be educated in what is known in mathematics. Finally, different groups of people, or organisations, may have influence in framing the questions that mathematicians are encouraged (and resourced) to explore.

But the same is true of invention. At different times and in different cultures there have been quite different views taken on the value of change, and thus invention. At some points in history the mainstream view has been that the crucial task is to preserve the known truth (for example, as discovered by some earlier civilisation - notably the ancient Greeks, or as stated in a holy book). A other times or places much greater value has been placed on inventing new knowledge. Even when invention is in good standing there can be a big question of who is to be permitted to do it. And even if invention is applauded it may be still true that this may only be in certain areas considered appropriate or important. This is as true in mathematics as in other areas of human activity.

In short, a lot of factors can shape what is seen as “mathematics”, what it is to be used for, and by whom. As an illustration it is worth remembering that astrology has until relatively recently been considered both a legitimate area of human knowledge and a key impetus for mathematical development. Thus E. G. Taylor writes of the understandings in England in the late sixteenth century: “The dictum that mathematics was evil for long cut at the very roots of the mathematical arts and practices. How were those to be answered for whom mathematics meant astronomy, astronomy meant astrology, astrology meant demonology, and demonology was demonstrably evil?"[6] Indeed, it was noted that when the first mathematical Chairs were established at Oxford University, parents kept their sons from attending let they be 'smutted with the Black Art'.[7] However, despite these negative connotations, practioners of "the dark arts" played a strong role in developing and refining instruments and methodologies for recording and predicting the movement of "star signs" as they moved across the celestial sphere.

One of the key features of the contemporary world is its high level of interconnection. In such a world it is easy to imagine that developments in "mathematics" which happen in one place will be known and built on almost simultaneously in another. Yet that is a very modern concept. In most of history the movement of information across space and time has been slow and very imperfect. So what at what one time has been discovered in one place may well have been forgotten a generation or two later, and unheard of in many other places. For this reason, amongst others already mentioned, talk of the evolution of mathematics as if it had a definite timetable, and a single direction is likely to be very misleading.

History of course relies on evidence. We can only know where and when innovations have occurred when evidence of them can be uncovered. Even the partial picture thus uncovered reveals a patchwork of developments in different directions. That is certainly a shadow of the whole complex pattern of discovery, invention, forgetting, and re-discovery which will have been shaped at different times by particular needs and constraints of different cultures, values, political structures, religions, and practices. In short, understanding the evolution of calculating machines is assisted by investigating it in the context of the evolution of mathematical thinking. But that is no simple picture. The history of developments in calculators and mathematics has been embroidered and shaped by the the social, political and economic circumstances in which they emerged. At times, mathematical developments have shaped developments in calculators, and and other times, the opposite has been true.

1.1.3 What is a calculator?

"Calculator" could be taken to mean a variety of things. It could be calculation 'app.' on a smart phone, a stand alone elctronic calculator from the 1970s, or the motorised and before that hand-cranked mechanical devices that preceded the electronic machines. In earlier times it could simply mean someone who calculates. It is difficult to see where the line should be drawn in this regress all the way back to the abstract manipulation of 'numbers'.

In this discussion, "calculator" is used as shorthand for "calculating technology". In particular it is taken to mean any physically embodied methodology, however basic, used to assist the performance of arithmetic operations (including counting). Thus a

6. E.G.R. Taylor, *The Mathematical Practitioners of Tudor & Stuart England 1485–1714*, Cambridge University Press for the Institute of Navigation, 1970, p. 4.
7. John Aubrey quoted in Taylor, ibid, p. 8.

set of stones laid out to show what the result is if two are added to three (to give five), or if in three identical rows of five what the outcome is of multiplying five by three (to give fifteen) will be regarded as a simple calculator. So too, will the fingers of the hand, when used for similar purpose, and even the marking of marks on a medium (such as sand, clay or papyrus) to achieve a similar result.

This approach is certainly not that taken in all the literature. Ernest Martin in his widely cited book *The Calculating Machines (Die Rechenmaschinen)* is at pains to argue of the abacus (as well as slide rules, and similar devices), that "it is erroneous to term this instrument a machine because it lacks the characteristics of a machine".[8] In deference to this what is referred to here is "calculators" (and sometimes "calculating technologies or "calculating devices"). Where the phrase "calculating machine" is used it will be in the sense used by Martin, referring to something with more than just a basic mechanism which would widely be understood to be a machine. But with that caveat, the term "calculator" will be used here very broadly.

The decision to apparently stretch the concept of calculator so far reflects a well known observation within the History and Philsophy of Science and Technology that in the end, technique and technology, or science and technology, are not completely distinct categories. Technologies embody knowledge, the development of technologies can press forward the boundaries of knowledge, and technological development is central to discovery in science. As Mayr says in one of many essays on the subject, "If we can make out boundaries at all between what we call science and technology, they are usually arbitrary."[9] Indeed, as will be described later, the mental image that mathematics is the work of mathematicians ('thinkers') whilst calculators are the work of artisans ('practical working people') is an attempt at a distinction that falls over historically, sociologically, and philosophically.

1.1.4 A discussion in three parts.

This book should be regarded as a work in progress. Corrections, additional insights, or links to other resources I should know about will be much appreciated.

A word also about the way I have constructed the historical account. In keeping with the analysis I have contributed to elsewhere (in a book by Joseph Camilleri and myself[10]), human development, will roughly be divided into a set of semi-distinct (but overlapping) epochs, preceded by a "pre-Modern Era" spanning the enormous time period from the birth of the first modern homo-sapiens to the beginning of the "Modern Period". This beginning is set as beginning (somewhat earlier than is

8. Ernest Martin, *The Calculating Machines (Die Rechenmaschinen)*, 1925, Translated and reprinted by Peggy Aldrich Kidwell and Michael R. Williams for the Charles Babbage Institute, Reprint Series for the History of Computing, Vol 16, MIT Press, Cambridge, Mass, 1992, p. 1.
9. Otto Mayr, "The science-technology relationship", in Barry Barnes and David Edge (eds), *Science in Context*, The MIT Press, Cambridge USA, 1982, p.157.
10. Joseph Camilleri and Jim Falk, *Worlds in Transition: Evolving Governance Across a Stressed Planet, Edward Elgar, UK, 2009*[11]

conventional) in the middle of the sixteenth century, with the “Early Modern Period” continuing from the mid-sixteenth to late eighteenth century, and the “Late Modern Period” stretching forward into the twentieth century, and terminating around the two world wars. From thereon the world is regarded by Joseph Camilleri and myself as entering a period of transition[12] (but there is not much need to focus on that here).

Thus the historical account is broken into three parts. The first part looks at the relationship between the evolution of calculating and calculators in the pre-Modern period. That forms a backdrop but only one object in the collection is of an appropriate age. Apart from that object (which is some 4,000 years old) the objects in this “collection Calculant” are drawn from the Modern Period (the earliest of these objects being from the early sixteenth century), and the Late Modern Period (from 1800) when mechanical calculation began to gain greater use in the broader society.

12. ibid, pp. 132–45.

Chapter 2

Origins

2.1 Part 1. Origins

2.1.1 Counting, numbers and counting technologies - did one come first?

Any account of the history of arithmetic (and it's big brother, mathematics) is to an extent thwarted by the fact that mathematical capacity, and almost certainly mathematical thinking, extend back before the time of recorded human history. There is by now growing evidence that some mathematical capacity is shared not only by humans but also by a range of animals, including monkeys and some fish and birds.[1] In short, some of our mathematical reasoning probably reflects an evolved endowment of the human brain. For that reason the axiom that equals subtracted from equals leaves equals (one of Euklid's "common opinions") is perhaps something we "know" rather than "arrive at".

While clearly there are capabilities in the human brain which enable quantity to be assessed, we need to resist the temptation to believe that particular mathematical capacities are 'hard wired' through evolution (and then make some retrospective argument about how that would have been good for survival). Some underlying capacities no doubt are innate, but determining which of them are is a difficult process. Indeed, a range of new highly revealing imaging technology supports a now widely held view within the field of neurophysiology that brains, including the human brain, are remarkably "plastic". That is they are self-organising, with the growth of neurone and connections between them being promoted by what we think about, and what we do. This capability has no doubt enabled creatures with brains, such as ourselves, to adapt more effectively to changes in our environment. This same capacity will to some extent allow the brain to "rewire" itself in a way that increases its capacity to reason mathematically. That will be shaped in part by the incentives in a society at a particular time for its members to be able to reason mathematically.

But this rewiring of our brains is in turn directed in part by the way we mould our context. That is shaped not the least by the technologies we create to assist us. For example, in the first half of the last century there was increasing social demand to be able to do mental arithmetic. So quite possibly we developed within our brains enhanced capacities to do that. With the advent of personal computing machines that capacity is less called upon. As has been pointed out forcefully more generally about computers and the internet,[2] from this point of view, calculators, as we create and use them, may be reshaping our brains and their capacities to do certain forms of mathematical and associated reasoning.

It may be useful to think of the emergence of counting in an unusual way (at least

1. see for example, papers cited in `http://www.scientificamerican.com/article.cfm?id=how-animals-have-the-ability-to-count`, Michael Tennesen, "More Animals Seem to Have Some Ability to Count", *Scientific American*, September 2009, viewed 19 Dec 2011.
2. Nicholas Carr, *The Shallows: What the Internet is Doing to Our Brains*, W. W. Norton, New York and London, 2010

in terms of the literature). First we may recall that in the approach to 'artificial intelligence' in which networks are set up using computers to mimic the neurological networks in the brain, these prove remarkably effective in pattern recognition.[3] The distinction between counting and pattern recognition can be fine indeed. For example, in one of numerous experiments carried out with animals, chicks when imprinted with five objects as constituting their 'mother' then searched for her when two of the objects were removed.[4] Similarly, recently it was reported that a neural network which had not been programmed with the concept of number was able to develop a capacity to identify patterns which had more dots.[5] Said the leader of the research, Marco Zorzi (University of Padua) "It answers the question of how numerosity emerges without teaching anything about numbers in the first place."[6]

Another clue to the long-standing nature of the required evolutionary endowments is found with experiments with baboons which were able to be taught not only to discriminate between four letter English words and nonsense combinations but also to achieve increased capacity to guess whether a word they had not seen before was an English word.[7] This shows the surprisingly sophisticated and relevant capacities of pattern recognition likely to have been possessed even by the human species' evolutionary ancestors.

It is possible therefore that the act of counting is one where an evolutionarily endowed physical capacity for pattern recognition is complemented by the capacity to manipulate objects (whether fingers, marks on bones, or counters) thus constructing abstract formulations of the pattern in relation to observed patterns. From this the invention of words to associate with the abstraction is but one additional step, and symbols as shorthand for those words, another. This raises the interesting question of whether the usual assumption that calculators were invented to assist counting should be reversed, with the possibility that the (very rudimentary) calculator was a necessary step towards developing counting.

One technological practice which is believed to have existed as long ago as the Upper Paleolithic period (40,000–10,000 years ago) in the region of Lower Austria (Moravia) and South Poland, is weaving. In excavations (dated as early as 35 thousand years ago) imprints of textiles have been found on the surface of some ceramic fragments.[8] Weaving certainly involves a capacity for pattern recognition,

3. Brian D. Ripley, *Pattern recognition and neural networks*, Cambridge University Press, UK, 1996
4. Tennesen, *Scientific American*, op. cit.
5. Ivilin Stoianov and Marco Zorzi, "Emergence of a 'visual number sense' in hierarchical generative models", *Nature Neuroscience*, advance online publication, Sunday, 8 January 2012/online, `http://dx.doi.org/10.1038/nn.2996` (viewed 21 Jan 2012)
6. Celeste Biever, "Neural network gets an idea of number without counting", *New Scientist*, Issue 2848, 20 Jan 2012.
7. Jonathan Grainger, Stéphane Dufau, Marie Montant, Johannes C. Ziegler, JoÃ«l Fagot, "Orthographic Processing in Baboons (Papio papio)", *Science*, Vol. 336 no. 6078, 13 April 2012, pp. 245–248.
8. J.M. Adovasio, O. Soffer, D.C. Hyland Textiles and cordage, Pavlov I – Southeast: A window into the Gravettian lifestyles, *Dol. Vest. Stud.* 14, Brno, 2005, p. 432–443, cited in `http://paleo.revues.org/index607.html` by Jiri A. Svoboda , "The Gravettian on the Middle Danube" *Paléo [En ligne]* , 19, 2007 , mis en ligne le 23 avril 2009, Consulté le 21 décembre 2011

and perhaps some concept of tracking the quantity of successive threads. Perhaps this is an early indication of the building blocks for mathematical thinking already in play.

There is by now evidence from both anthropological and psychological research relating both to the oral presence of numbers in different societies and the presence of written words or symbols for them. Indeed this has led to the emerging field of ethnomathematics. However, the conclusions are not clear cut. We may simply all share some basic capacity to do counting and mathematical thinking. But what is innate in this way is hard to pin down. In any case, whether and how that capacity is taken up and developed will depend on the cultural and historical circumstances and needs of a culture. Indeed, whilst there are differences in what we recognise as mathematical cognitive abilities in different societies it seems that these differences cannot be taken to "imply necessary distinctions between right/wrong, simple/complex or primitive/evolved."[9]

There is plenty of archeological evidence that the capacity to count is very ancient. Boyer and Merzbach suggest that it came about first through recognition of sameness and difference, and then over time the recognition that collections of things with sameness can be given a short description which we now call number. They suggest this process was probably very gradual and may have evolved very early in human development, perhaps some 300,000 years ago - about the same time as the first known use of fire.[10] However they are also quick to note that whilst we may make conjectures about the origins of the concept of counting, since counting emerged prior to the earliest civilisations and certainly before written records, "to categorically identify a specific origin in space or time, is to mistake conjecture for history."[11]

Nevertheless, there is evidence that the idea of associating things to be counted with a set of abstract counters is long standing. A Baboon bone dated from 35,000 years ago (amongst others of similarly great age) has been found with what are believed to be tally marks scribed on it.[12]) Another more recent bone, from about 25,000 years ago (see in figure 2.1 below) found in the village of Ishango at one of the farthest reaches of the Nile, has a much more complex set of notches which may be calculating tables, but is probably a crude lunar calendar.[13]

The prevalence of five and ten based counting systems in the most ancient surviving records suggests that the fingers also have long been used as a handy, although not universal, set of counters. (The Kewa people of Papua New Guinea are reported to count from 1 to 68 on different parts of their bodies.)[14] Australian Aboriginal

9. Stephen Chrisomalis, "The cognitive and cultural foundations of numbers", *Oxford Handbook*, pp. 495–502.
10. Carl B. Boyer and Uta C. Merzbach, *History of Mathematics*, Wiley, 2010, p. 2.
11. ibid p. 7.
12. Jonas Bogoshi, Kevin Naidoo, John Webb, The Oldest Mathematical Artefact, *The Mathematical Gazette*, Vol. 71, No. 458 (Dec., 1987), p. 294.
13. D. Huylebrouck, "The Bone that Began the Space Odyssey", *The Mathematical Intelligencer*, Vol. 18, No. 4, pp. 56–60
14. Frank J. Swetz, "Bodily Mathematics" in *From Five Fingers to Infinity: A Journey through the History of Mathematics*, Open Court, Chicago, 1994, p. 52.

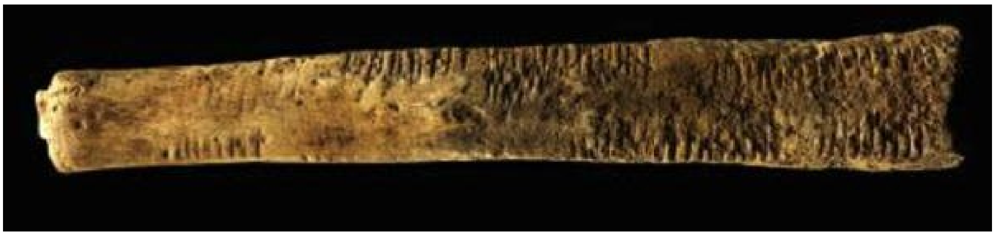


Figure 2.1. Ishango Bone - a small piece of Baboon fibula marked with 3 rows of well defined tally-mark notches ~25,000 BC. Reproduced courtesy of the the Smithsonian Institution Museum of Natural History (under its terms of use).

and Islander groups, contrary to earlier reports that they had limited number systems, have been shown to be able to count with facility, not necessarily by reciting the words for numbers, but by subitizing - relating the numerosity of a set of objects to grouped 'hands' of fives.[15] The earliest counting, it has been suggested, may likely have been with pebbles which were conveniently at hand. But even though this seems credible, their use in this way may prove illusive to discovery through contemporary archeology.[16]

There may be a temptation to suggest that the earliest surviving instance or record of a calculator constitutes the moment that the technology of calculation first emerged. But as the above suggests, this is a crass simplification. Even beyond fingers and stones the earliest approaches may have been made using scratches in the dust, or fragile organic materials (such as leaves or pieces of grass), which except perhaps in very dry graves, would be unlikely to stay the distance. Thus for example we do not know the many possible ways early devices such as the knotted string khipu[17] of the Incas was used. We do know that this device, always composed of many strands of knotted strings, but with great diversity in its use, not only represented a decimal, double entry accounting system, but also was used in functions of state. They ranged from recording outcomes of the national census as carried out district by district, and then compiled nationally, to the calculation of tributes, culturally significant astronomical events, and much more.[18] A surviving example is shown in figure 2.2 below.[19]

The earliest surviving tokens recognised by archaeologists as being for counting and (primitive) accounting can be found from 8,000 BC in the remains of Neolithic settlements. This was a time of early deployment of agricultural practices, in what is now Syria and Iran. Success in agriculture could be enhanced by record keeping as

15. John Harris, "Australian Aboriginal and Islander mathematics", *Australian Aboriginal Studies*, number 2, 1987, pp. 29–37.
16. Denise Schmandt-Besserat, "The Origins of Writing: An Archaeologist's Perspective", *Written Communication*, Vol 3, 1986, p. 35.
17. See, for example, `http://khipukamayuq.fas.harvard.edu`
18. Gary Urton, "Mathematics and Authority: a case study in Old and New World Accounting", in Robson and Stedall, *The Oxford handbook*, p. 34–49.
19. From the Museum for World Culture, Göteborg, Sweden. Image retained in the Harvard University Khipu Database at khipukamayuq.fas.harvard.edu/images/KhipuGallery/MiscAlbum/images/UR113%20Valhalla_jpg.jpg (viewed 27 Dec 2011)

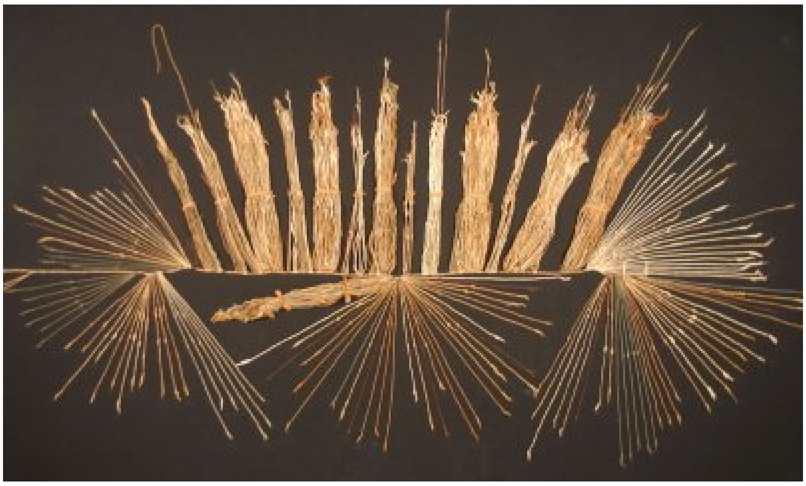


Figure 2.2. Inca Khipu - made of 322 strands and said to be from Nosca, Peru. Museum of World Cultures, Göteborg, Sweden (photo courtesy of Gary Urton)

well as exchange. This was now appropriate since settlement enabled an increasingly sophisticated division of labour to emerge. Over the next five thousand years (to 3,100 BC) these artifacts can be seen to evolve to tokens (essentially pebbles fashioned from clay but with different shapes to connote different things, such as a disk representing one animal, or a cone representing a quantity of grain). Examples are shown in figure 2.3 below.[20]

These tokens were in time enclosed in clay envelopes holding tokens of particular transactions strung together on strings. Envelopes, however, hid the enclosed tokens and so envelopes emerged bearing images of the contained tokens impressed on their surfaces. Later clay envelopes can be found with signs not merely impressed upon them but also scribed into them.[21] For an example see figure 2.4 below.

As Schmandt-Besserat points out in her important, although not uncontested account,[22] "The substitution of signs for tokens was no less than the invention of writing."[23] This supports the observation, made by several authors,[24] but developed in considerable detail by Schmandt-Besserat that these inscriptions not only preceded

20. Denise Schmandt-Besserat, "The Origins of Writing : An Archaeologist's Perspective", *Written Communication* Vol 3, No1, January, 1986, Figure 3, p. 41. For more of the work of this eminent author see Denise Schmandt-Besserat (ed.), *Symbols at 'Ain Ghazal*, in *Ghazal Excavation Reports 3*, bibliotheca neolithica Asiae meridionalis et occidentals, 2013, & Yarmouk University, Monograph of the Faculty of Archaeology and Anthropology, 2013; Denise Schmandt-Besserat , *Before Writing: From writing to cuneiform*, The University of Texas Press, USA, 1992, and Denise Schmandt-Besserat, *How Writing came about*, University of Texas Press, USA, 1996.
21. Denise Schmandt-Besserat, "The Envelopes That Bear the First Writing", *Technology and Culture*, Vol. 21, No. 3, July 1980, Figure 1, p. 358.
22. see for example, Zimansky, "Review of Schmandt-Besserat 1992", in *Journal of Field Archaeology*, Vol. 20, 1993, pp. 513–7.
23. Schmandt-Besserat, *Written Communication*, 1986, p. 37.
24. for example, Livio C. Stecchini, "The Origin of the Alphabet", *American Behavioral Scientist*, Vol 4, February, 1961, pp. 3–7

Figure 2.3. Complex Tokens - from Susa, present day Iran, ~3300 BC. Below from left to right: one sheep, one unit of textile, 1 unit of honey, one jar of oil. Above from left to right: one ewe, ?, one ingot of metal. The objects are kept at the Musée du Louvre, département des Antiquités Orientales, Paris, France. Courtesy Denise Schmandt-Besserat.

Figure 2.4. Impressed envelope- with its token content from Susa, present day Iran, ~3300 BC. The large and small cones are units of grain. Each of the lenticular disks stand for 10 sheep. The objects are kept at the Musée du Louvre, département des Antiquités Orientales, Paris, France. Courtesy Denise Schmandt-Besserat.

the appearance of the first known written alphabet (cuneiform) but also appear to prefigure it. This lays a basis for the intriguing proposition that rather than writing being the basis for mathematics, the primitive technologies of calculating (and the mathematics that underlies it) may have not only preceded but formed the basis for the development of the first written scripts upon which has been built the technology of writing.

The Sumerian civilisation, as already mentioned, was the source of cuneiform script, the earliest known alphabetic writing system. In the period prior to 3,500 BC in the fertile plain of Mesopotamia between the Tigris and Euphrates rivers an advanced system of settlements, agriculture, irrigation and social organisation provided an equally fertile environment for the invention and widespread use of cuneiform. The number system developed within this script was based on powers of sixty rather than powers of ten as in contemporary systems. Even so, these 'sexagesimal' numbers[25] were constructed with patterns corresponding to the numbers from 1 to 10. During the period of Akkadian rule, which lasted to 2100 BC, the abacus entered Sumerian life creating a further extension to the capacity to form basic arithmetic operations. The Babylonian civilisation replaced that of the Sumerians around 2000 BC.

Whilst the Babylonian civilisation replaced that of the Sumerians there was continuity in the specialisation of roles in the community, the reliance on trade, and thus a value on numerical script. The object shown below (table 2.1, *Old Babylonian Tag*) is an Old Babylonian tag receipt for animal carcasses including 1 ewe and 1 ram, from between 1934–1924 BC, and is covered in the administrative script of the time - numbers, alphabetic characters, and the imprints of official seals. Originally on a string the seal indicates the receiving party was important, since the seal legend was dedicated to a king, perhaps of Isin.[26]

Large numbers of cuneiform tablets found in archaeological digs reveal that by now a mathematical system, recognisable to modern sensibilities, had emerged. This went hand in hand with the development of an organised urbanised agricultural society achieving significant construction (especially of canals). The day was divided into 24 hours, the hour into 60 minutes, the minutes into 60 seconds, and the circle was divided into 360 degrees. All of this sexagessimal flavour persists to the present. The tablets (for example, see figure 2.5 below[27]) showed now the construction of reference tables to aid calculation including squares of numbers, tables of reciprocals to aid division, and more.

A basic form of algebra had also been developed, with equations and solutions, including solutions to quadratic equations that arose in the course of engineering of canals and other structures.[29] Schools were also established so that the knowledge

25. See, for example, `http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Babylonian_numerals.html`
26. Translation thanks to Bob Englund, private communication, 18 March 2014, who stresses that it needs further interpretation by an expert in Old Babylonian administrative script.
27. Eleanor Robson, "Mathematical Education in an Old Babylonian Scribal School", in Robson and Stedall, *The Oxford handbook*, Figure 3.1.4, p. 210.
29. ibid

Table 2.1. Old Babylonian Tag

Babylonian tag receipt for animal carcasses including 1 ewe and 1 ram, 1934–1924 BC (collection Calculant)

required to read and write cuneiform, and perform mathematical operations using it, could be transmitted.[30] In this sense, now recognisably analogous to modern writing and media, the cuneiform tablets, combined with a social order which both needed them, and trained in their use, more than 3,000 years ago had emerged as a socially powerful mathematical and scribal technology.

The idea of numerals to represent numbers diffused and developed over following centuries emerging in different representations in different places. True to the importance of the human hand, most of these systems privileged the number 5 and 10, with 10 emerging as the most common "base" the powers of which shaped the meaning of successive positions in a string of numerals. Two different innovations should be distinguished here. The first is to develop numerals corresponding with successive quantities. The second is to develop a "place value" system of writing down numerals. In such a system the place a numeral occupies indicates that the corresponding number is multiplied by a power (determined by the placing of the numeral) of the base . (For example, in modern script, whose base is 10, and whose places from the right indicate "units", "tens", "hundreds", etc, the number 123 represents 1×10×10 + 2×10 + 1×1).

It is unwise to assume that the history of counting, numbers, and indeed script has

30. Eleanor Robson, "Mathematics education in an Old Babylonian scribal school", in Robson and Stedall, *The Oxford handbook*, pp. 209–227

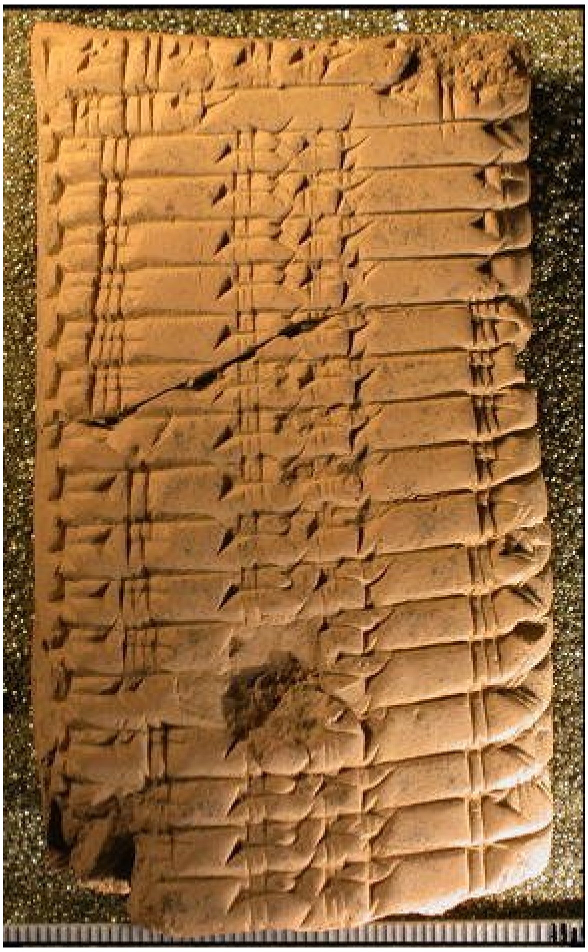


Figure 2.5. Babylonian scribal tablet showing list of reciprocals ~1800 BC. MS 3874 Friberg, A Remarkable Collection (2007), 69. Courtesy of Jöran Friberg.[28]

a single line of development. Stephen Chrisomalis cautions us, it is highly probable that "the modern place value numerical notation, or something quite like it, developed at least five times idependently" - in: Mesopotania (as already mentioned ~2100 BC), China in ~14–1300 BC, India in ~500 AD, and the Andes in or before 1300 AD, with the explanation that it is not as big a cognitive leap to develop such a system when it proves useful as is often suggested.[31] To put it another way, some combination of our biologically endowed cognitive capacities and underlying evolved cultural building blocks may be conducive to assembling quantities in this way.

Some 100 different scripts have been identified which have emerged over the last five millennia.[32] Of these numeric scripts, however, the earliest dated are the Proto-Cuniform (already discussed) and the ancient Egyptian. The ancient Egyptians

31. Chrisomalis, Oxford Handbook, p. 509.
32. Stephen Chrisomalis, "A Cognitive Typology for Numerical Notation", *Cambridge Archaeological Journal* Vol 14, Issue 1, 2004, p. 37.

developed a number system which was different in the base (this time 10 rather than 60 in cuneiform) and in the characters used. The ancient Egyptian system of hieroglyphic numerals, developed as early as 3250 BC,[33] had characters for 1 and then the powers of 10 (10 - a vertical stroke, 100 - an inverted wicket, 1000 a snare, etc.) with the numbers from 1 to 9 simply shown as the corresponding repetition of the number 1. Thus for example, the number 12345 would appear as .[34] However, after about a millennium of use of this system another "Hieratic" script was developed for use on Papyrus for routine use as opposed to the hieroglyphic script which was retained for carving in rock. The Hieratic numerals (shown in the table below) had by now taken what to modern eyes is the more efficient form of single symbol "ciphers" to represent each of the integers from 1 to 10 - the same concept which forms the basis for the modern numerals in use today.

As shown in table 2.2, *Evolving Number Systems*, below, other early representations similarly drew directly on patterns representing counters (or fingers). (Even the Roman system can be seen as counting to five on the one hand, reserving the thumb and forefinger for the V to represent five, and the X representing a V on each hand). The Egyptian Hieratic and then the Greek system replaced combination numerals with single characters, and finally, from the eighth century, the familiar symbols of the modern (Arabic-Indian) place-based system (complete with the numeral 0 to replace earlier spaces for "place holders" finally emerged.[35]

Table 2.2. Evolving Number Systems

Number System	Script[42]	Base	~ Century Introduced[43]
Proto-Cuneiform		60	3200 BC
Egyptian Hieroglyphic[44]		10	3200 BC
Egyptian Hieratic[45]		10	2600 BC
Greek	α β γ δ ε ϝ ζ η θ ι	10	575 BC
Roman	I II III IV V VI VII VIII IX X	10	500 BC
Chinese Rod		10	300 BC
Indian[46] C8 AD		10	700 AD
Arabic[47] C11 AD		10	1000 AD
European[48] (Arabic-Indian) C15 AD		10	1400 AD
Modern (Arabic-Indian) C16 AD[49]	1 2 3 4 5 6 7 8 9 0	10	1549 AD

33. ibid, p. 51.
34. characters reproduced from Boyer and Merzbach, *History of Mathematics*, p. 10
35. For a more detailed account of the multiple scripts which emerged and a classification of them see Georges Ifrah, *The Universal History of Computing: From the Abacus to the Quantum Computer*, John Wiley & Sons, USA, 2001, pp. 26–63.

2.1.2 Numerals, counting and counting devices - a symbiotic relationship

It is fairly easy to see how additional counting devices might evolve from the earlier primitive counting technologies. Most obviously marks, pebbles and tokens, and then grouped tokens, some of them strung like beads, lead fairly naturally to more efficiently arranged arrays of counters or special purpose rods, whether laid out on a backing, or strung along the lines of a primitive weaving frame.

Early devices include the development of knotted ropes used for both measurement and arithmetic operations. For example, two knotted ropes end on end may give the addition of two numbers. A knotted rope whose ends are brought together will provide a measure of half the original number, and so on. Further knotted ropes may be used to develop geometric relationships (for example, a 3–4–5 triangle can be used to set a right angle).[50] Knotted ropes (see below[51]) were used (by "rope stretchers") for measurement in ancient Egypt (and perhaps mathematical operations) - see figure 2.6 below. Use of knotted ropes in ancient China is referred to wistfully by philosopher Lao-tze in the sixth century BC when he asks "Let the people return to the knotted cords and use them."[52]

Figure 2.6. Rope stretchers - measuring the land for agriculture. Picture in the Tomb Chapel of Menna, Luxor (Thebes) ~1200 BC. Photo courtesy of the Musée canadien des civilisations

The Chinese are known to have moved from knotted ropes to a system of counting rods (see figure 2.7 below),[53] becoming very proficient in their use. Perhaps the

50. There is much debate in the literature about whether the Egyptians used this property in the construction of the Pyramids. In sum there is probably no evidence that they did, but some evidence they may have used it in the construction of tombs. The only firm evidence that this relationship was known and taught is from a Demotic papyrus from the third century BC. There is earlier suggestive evidence. See Corinna Rossi, *Architecture and Mathematics in Ancient Egypt*, Cambridge University Press, Cambridge, UK,2004, p. 9.
51. Measuring the land using rope, from the Tomb Chapel of Menna, New Kingdom, Valley of the Nobles, Thebes, Egypt, contained in the Bridgeman Art Library, reproduced at various sites on the web including civilization.ca/civil/egypt/images/fback3b.jpg (viewed 26 Dec 2011) and with fuller description at gettyimages.com.au/detail/illustration/measuring-the-land-using-rope-from-the-tomb-chapel-of-stock-graphic/55995332
52. Loa-tze, Tao-teh-king, English Edition, P. Carus (ed.), Chicago, 1898, pp. 137, 272, 323, cited in David Eugene Smith and Yoshio Mikami, *A History of Japanese Mathematics*, The Open Court Publishing Company, Chicago, 1914, p. 20
53. at kaleidoscope.cultural-china.com/en/137K6K6434.html|Cultural China (viewed 28 Dec 2011)

earliest written account of the use of these comes from the Han Shu records of the Han Dynasty written by Pan Ku in 80 AD who relates that the ancient Chinese used sets of 270 rods to perform arithmetic calculations..[54]

Figure 2.7. Metal counting rods - Western Han Dynasty, unearthed in Xi'an of Shaanxi Province ~0–200 BC

The rod numerals in the table of numerals shown earlier (and described by philsopher Ts'ai Ch'en (1167–1230 AD) give some indication of how these counting rods might have been used. Sun-tsu in the Third Century AD writes that the units should be vertical, the tens horizontal, the hundreds vertical and so on, and that single rod may suffice for 5. The results of a multiplication of 247 x 736 is given in this system by Yang Houei in about 1276. Such rods, made of bamboo, were known to have been in use in Japan by the seventh century AD, and were later replaced by more stylised "sangi pieces" - square prisms about 7 mm thick and 5 cm long and reasonably extensive records of calculations using chess board like "swan-pan" or "sangi boards" survive from the seventeenth century (see figure 2.8 below[55]). Similarly, rods, made of bamboo and numbering 150 in a set, are still used in Korea.[56]

The above innovations can be seen to fairly easily give rise to the emergence of different forms of counting machines. Of these the most well known surviving example in its multiple guises is the abacus. The invention of the abacus is attributed by some to the Akkadians who invaded the Sumerian civilisation around 2300 BC.[57]

However, what is to be taken to constitute an abacus has fuzzy boundaries. Arguably

54. Smith and Mikami, *A History of Japanese Mathematics*, p. 20.
55. From Mayake Kenryu's work of 1795, reproduced in Smith and Mikami, *A History of Japanese Mathematics*, p. 29.
56. ibd, p. 21
57. see for example, `http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Babylonian_mathematics.html`, "An overview of Babylonian mathematics", viewed 21 December 2011

Figure 2.8. Swan-pan board - being used for calculation

it has taken different forms as it emerged in different places at different times.[58] Between pebbles on the ground and the abacus can be taken to lie counting rods (as developed by the ancient Chinese), and counting boards with scribed or otherwise arranged positions for counters. The word “abacus” is said to derive from the Semitic word abaq for dust, perhaps indicating that it developed from a sand tray used for counting. Latin (abakos) and Greek (αβαξ) versions of the word followed.[59] There is reference by the ancient Greek historian Herodotus in the fifth century BC to hand movements where the Egyptians move from right to left in counting, whilst the Greeks move left to right, suggesting perhaps the operation of some counting frame or board.[60]) There is what appears to be a surviving marble counting board dated at around the fourth century BC in the National Museum in Athens, see figure 2.9 below.[61]

Archaelogical evidence exists of the Roman embodiment of the abacus. Forms of it range from a wax tablet for scribing, a metal plate with sliders of which one of the two surviving examples is shown in figure 2.10 below.[62] It is held at the Museo Archeologico Reionale of the Regione autonomy Valle d’Aosta, Greece, where it was discovered in a tomb along with other grave goods. As has noted by Searfimo Cuomo,[63] since grave goods were both indicators of status and also were connected to, or possessions of the deceased, it raises the possibility that this was the grave of a professional ‘calculator’. An image of another such abacus held by the Museo

58. For some nice images of the various types of abacus which emerged from different cultures see `http://en.wikipedia.org/wiki/Abacus`, the coresponding Wikpedia article (viewed 23 Dec 2011)
59. ibid
60. Boyer and Merzbach, *History of Mathematics*, pp. 179–80.
61. The oldest surviving counting board, made of marble. Photo from the National Museum of Epigraphy, Athens, reproduced from Menninger, *A Cultural History of Numbers*, Fig. 128, p. 300.
62. This image is adapted from a photography by Mark Cartwright (`http://www.ancient.eu.com/image/2147/`)
63. `http://www.gresham.ac.uk/lectures-and-events/early-mathematics-day`

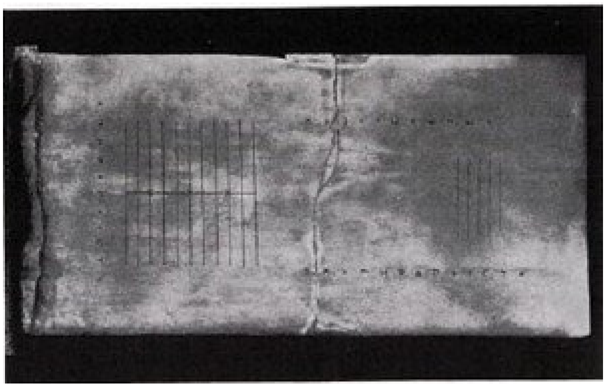


Figure 2.9. Salamis Tablet

Nazionale Ramano at Piazzi delle Terme, Rome is reproduced elsewhere.[64] Made of brass plate it is approximately 115 x 99 mm with nine long slots (equipped with four sliders for 1–4) and eight short slots each with one slider (for 5). The eight left most columns run in powers of ten from units on the right to millions on the left. The ninth slot is for a fraction of ounces. Further details are presented by Rossi et al.[65] Other embodiments included a grooved counting board, or simply a table on which counters could be moved.[66]

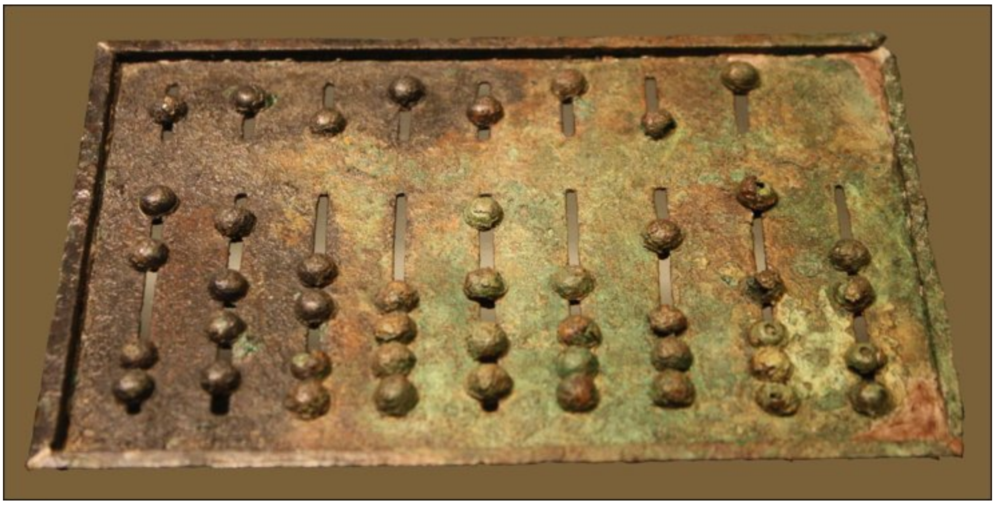


Figure 2.10. Roman abacus. On display in the Museo Archeologico Regionale, Aosta, Greece. Photo courtesy Carlos Dorce Polo.

64. `http://www.ee.ryerson.ca/~elf/abacus/roman-hand-abacus.html`
65. Cesare Rossi, Flavio Russo and Ferrucio Russo, "Ancient Engineers and Inventions, *History of Mechanism and Machine Science*, Springer, Vol. 8, 2009, p. 42
66. see Karl Menninger, *A Cultural History of Numbers*, Dover, 1992 (republication of MIT Press English Translation, 1969, of the German Edition, Zahlwot und Ziffer: Eine Kulturgeschichte der Zahlen, Vanderhoek & Ruprecht, Germany, 1957–1958); and more generally `http://en.wikipedia.org/wiki/Roman_abacus` - the corresponding Wikpedia article. (viewed 23 Dec 2011)

The abacus remains a highly efficient calculating device in widespread use across Asia and Africa. The emerging Arabic abacus was simply constructed of rows of wires bearing ten balls each, as still does the Russian abacus ("schoty"). The columns of the Chinese abacus (the "suan pan") are divided into two sets of rows of beads, the upper ones each representing five on the lower section (see figure 2.11 below.

Figure 2.11. Chinese Abacus(collection Calculant)

The Japanese abacus ("soroban") has gone through a transition from the Chinese form (which arrived in Japan in about the 17th century AD), to a simplified form commencing about 1850 with only one bead in the upper, and five in the lower section, to a form nationally standardised in 1944 to only one bead in the upper, and four beads in the lower section for each column.[67] In this sense it moved through various stages of development back to the configuration of the Roman abacus of two thousand years before.[68]

The symbiotic relationship between number systems and counting devices can be seen very clearly in the evolution of the abacus on the one hand, and the persistent use of Roman numerals right into the middle ages in Europe. The importance of counting technology to supplement such systems can be illustrated as follows:

> Put down 3 pebbles and then put next to them 4 more. We can now count them and find we have 7. They have been added even though we have not consciously performed the mental act of addition let alone labelled it

67. James Cusick, "The Japanese Soroban: A Brief History and Comments on its Role", `http://www.jamescusick.net/pages/hosdocs/Cusick_SorobanHistory.pdf` (viewed 11 June 2011)
68. see for example, `http://sites.google.com/site/osakasoroban/news/soroban`|Osaka Abacus Association, "Soroban: The first calculator" (viewed 23 Dec 20110.

> as such. But we have invented the operation of adding (whatever we call it) as soon as we start using this to keep tally. We can do this whatever the number system. Count out III (3) pebbles (I, II, III). Add IV (4) more. Count out the result. We now can be seen to have VII (7).
>
> Arrange the pebbles in two columns on a board, so that when we get to 10 in one column we put one in the next column and put aside those already used. Then we have a positionally based counting system quite equivalent to modern notation, and we are adding decimally. Add 3 pebbles three times to make up a column of nine. The outcome is the equivalent of multiplying III x III or 3 x 3 to give IX (9). It only remains to give this sort of activity the name multiplication.
>
> Systematically perform these multiplications from I to X (1 to 10), writing down the outcomes, and we have a multiplication table, which can be learned in Roman numerals, or in modern numerals, and in any case is usually learned by the names for the numbers rather than the numerals ('two twos are four" or *perhaps* in Latin "duo et duo quatuor ponit").
>
> With the pebbles standardised for ease of use, and arranged on pre-set columns (grooves, wires, lines), we have a device which enables the arithmetic operations to be performed with relative ease in whichever numerals are used. (Division is harder, but perfectly possible, although it is probably this activity which in the Middle Ages gave rise to the famous description of "the sweating abacist".[69])

This demonstrates the way in which counters, counting boards and rods, and eventually the abacus performed the essential duty of translation between pre-Indian numerals and the tasks of arithmetic. As Karl Menninger points out, this method was so effective that there was enormous reluctance to give up the old scripts even when the more efficient single symbol Indian-Arabic scripts were available. As he put it "The mutually complementary use of numerals and the counting board thus created a fully adequate and convenient tool for simple computation, which people were extremely reluctant to part with... Not only did Medieval Europe possess it (the modern place value notation) for many centuries, but it was throughly familiar to people even in antiquity - on the counting board." But "It never occurred to anyone to try to take the step that the Indians had taken".[70]

One could argue that the efficiency of the abacus is so great that there was no purpose in adopting the Indian script. For example, on 12 November 1946, in a competition overseen by the US Army Newspaper, between a selected expert practioner of the latest electric calculating machine and an expert soroban practioner the soroban practitioner defeated his opponent 4–1 in the tests of multiplication, division, addition

69. "regulae quae a sudantibus abacistis vix intelleguntur" - "rules which the sweating abacists scarcely understand", quoted in Menninger, *A Cultural History of Numbers*, p. 327
70. Menninger, *A Cultural History of Numbers*, p. 298 et seq.

and subtraction. Declared a report in the *Nippon Times* "Civilization, on the threshold of the atomic age, tottered Monday afternoon as the 2,000-year-old abacus beat the electric calculating machine in adding, subtracting, dividing and a problem including all three with multiplication thrown in, according to UP. Only in multiplication alone did the machine triumph..."[71] [72]

However, the matter cannot be left there, for the use of counting boards and the abacus was also framed by the available media in which counting might be recorded. Metal and stone were used for writing in the early centuries in China. Clay was utilised by the ancient Sumerians. A much more tractable technology, papyrus, had been well used for a millenium in ancient Egypt but was unknown in ancient Greece before 700 BC. Parchment was invented around 400 BC. Paper came much later. It has been argued that the combined factors of cumbersome numerals, and difficult to use writing media, created a strong pressure to develop other technologies, such as the abacus, to complement them.[73]

Thus whilst the power of the abacus is indeed great - in highly trained hands - the social need for more widespread arithmetic capabilities in an ever more numerically ordered economcy, the cheap availability of paper, and the advent of printing and improved writing technology, made the capacity to calculate on the page, without any intervening calculating device, increasingly valuable. That could indeed be recognised as made much easier by an efficient positional decimal script. Thus the growing commercial pressure for wider arithmetic literacy in Europe was probably a factor in the adoption of the efficient Arabic-Indian script and abandonment of the abacus and counting board. (Much later, in the current period, this need for even wider basic mathematical literacy literacy would also be facilitated by cheap and freely available electronic calculators. The need for that, and battle to devise it, is a much later part of this story.)

To summarise, the development of systems of counting, technological modes of facilitating them, and particular social systems have co-evolved hand in hand. The process has often been quite slow and incremental. New ideas have not necessarily displaced old ones in practice for very long periods of time, if ever. And different evolutionary trajectories have developed in different social, economic and historical settings. But this insight is not restricted to the co-evolution of counting, numeration and supporting technologies. This incremental process of development has played out in different ancient societies not only in the business of arithmetic, but also in the development of broader mathematical concepts.

71. See `http://www.ee.ryerson.ca/~elf/abacus/abacus-contest.html`, "The Abacus vs. the Electric Calculator" (viewed 28 Dec 2011).
72. See also `http://www.britishpathe.com/record.php?id=44628`lsee also newsreel footage of a similar competition in Hong Kong in 1967 (viewed 28 Dec 2011).
73. Smith and Mikami, *A History of Japanese Mathematics*, p. 19; and a similar argument from Boyer and Merzbach, *History of Mathematics*, p. 228

2.1.3 Calculating technologies, and the evolution of mathematics

The above has focussed on the evolution of counting and the technologies of that. Yet, as already suggested in the discussion above, there is a seamless overlap between counting, and the broader fields of arithmetic and mathematics. One observation which is suggested, not always spelt out in the literature, is that a mathematical idea or artifact which appears to have been "invented" in a single leap of inspiration will more likely have evolved very gradually. The appearance of sudden invention is not unlike the "missing link" between baboon and human which used to be considered a problem for the theory of evolution. Now sufficient of such links have been found to support the theory. But evidence for outmoded ideas are less available (which is one of the charms of calculators which do leave a more enduring evolutionary trail). Nevertheless, it is not hard to see how the use of counters (whether fingers, pebbles, rods, knots or beads) as proxies for specific things (sheep, tenants, corn bushels) leads beyond counting, seamlessly to addition and subtraction, and then to more sophisticated mathematical ideas.

There is an enormous literature on the history of mathematics. By now it covers many societies beyond the recognised roots of Western society in Greece, Rome and Egypt. There is of course a rich history of Arabic Islamic mathematics which is still only beginning to be recognised in the "West", and beyond that the mathematical developments from societies ranging from the Inca, Indian, South American, to many surviving cultures of indigenous peoples. In this sense there is no single history of mathematics. More challengingly, there can be more than one "mathematics", a word which itself derives from ancient Greece, but in contrast to current usage had a much broader meaning of "learning".[74]

One seemingly quite general proposition which can reasonably be formulated from this literature is: pick any pre-modern society which has established sufficient record to be able to display its developments in the area of numerical and mathematical culture. There we will find practices of not only counting and basic arithmetic, but also invention and use of more advanced mathematical concepts. In each society, however, the particular sorts of emphases, consequent areas of discovery, and the way these are arrived at and formulated, may differ greatly. Further, the technologies used (whether, for example, use of diagrams, the abacus, or knots) will depend on the physical and social circumstances of the society. Here it is sufficient to illustrate this in the light of a few examples and make a number of observations about them.

Pragmatic mathematics - Mesopotanian, Egyptian, Chinese and Indian foundations

Mathematics has many expressions. But the evidence suggests, not surprisingly, that the beginnings were built by the people who needed to answer practical problems demanded by an increasingly sophisticated society. Examples include: how to build

74. G.E.R Lloyd "What was mathematics in the ancient world?", in Robson and Stedall, *The Oxford handbook*, p. 8.

a regular shape (for example, a pyramid) and how many people would be needed, how much food would they need, how much tribute would be required to keep the administration in operation, what would be the crop production and how much could each person give? The people who needed this sort of mathematics were scribes and architects, builders, and those who supervised the payment of tributes. There were also others, who we will occasionally refer to here (not quite comfortably) as "artisans" (in the sense of skilled worker) or "practitioner" (in the sense of a pragmatic practitioner or practical mathematics). The beginnings of mathematics can be found in ancient Egypt and Mesopotania, derived pragmatically from experience and shaped and written down precisely by and for such practitioners (including officials).

Egypt The role of environment on the form of mathematical development is famously illustrated by the case of ancient Egypt. Here the use of rope and rod measuring, and associated calculations and geometrical insights was essential to the calculation and arbitration of claims in relation to ownership of land after the regular Nile flooding. The Greek historian Heroditus after visiting Egypt claimed in about 450 BC that the origin of geometry (whose original meaning was "land measurement") was Egypt.[75] However, the idea that ancient Egypt had developed an elaborate discpline of geometry needs to be tempered by the character of mathematics which the Egyptians in fact created. As elsewhere this was shaped very much by the process of discovery itself, the culture, practices and challenges faced by the society, and as a consequence, the uses that society put mathematics to.

In Egypt, life was built around the fertile area of the Nile. Its periodic floods both laid down new soil and washed away salinity, but at the same time reshaped the land. There was a need for landholders to establish the boundaries of their land after this flooding and developing a means of proving those boundaries was an important motivation for the considerable advances in land measurement with knotted ropes and measuring sticks which can be found as far back as the records stretch. Over three thousand years of continuous civilisation gave rise to a complex society ruled through the sophisticated hierarchical dynastic governance of the Pharos. Accompanying that was the emergence of associated religious institutions (building legitimacy in part by a capacity for astronomical observation and prediction), complex economic taxation and trade relationships, and mining and fashioning of metal, and art, engineering and architecture (notably visible in the surviving pyramids, tombs and statues).

The extensive literature on ancient Egyptian mathematics needs to be read with the fact in mind that, in reality, the surviving written sources on the subject are very sparse. (In sum they are constituted from five papyri, a leather roll and two wooden tablets from 2055–1650 BC). Much of the historical attention focusses on the Rhind Papyrus, which appears to have been an instruction text for scribes, copied by scribe Ahmes

75. Corinna Rossi, "MIxing, building and feeding: mathematics and technology in ancient Egypt", in Robson and Stedall, *The Oxford handbook*, p. 418

in 1650–1550 BC from an older text from 1985–1795 BC.[76] The papyrus presented a series of problems and worked solutions of the sort that scribes might have to deal with in their various roles, but always in the form of specific case examples, then developed with increasing difficulty.

Evidence from the above papyri is complemented from information which may be induced from surviving technological artifacts - structures, objects in tombs, and inscriptions on tomb walls. From this corpus it is possible to draw quite a wide range of conclusions.[77] Here it is sufficient to note that the ancient Egyptians had developed a capacity to solve mathematical problems firmly rooted in the practical needs of the society. These needs included such things as calculating the area of land, the fraction of crops required in annual payment of dues to the state, the quality of products such as beer and bread (expressed as the “psw” or fraction of grain required to produce them), and quantities of ores and other ingredients to smelt metals. They knew about the practical measures required to measure out fields, and volumes of various shaped objects, and how to characterise a slope (for example of a pyramid by calculating the “sqd” - the distance, in number of palms, by which it deviated from the vertical in a vertical rise of one cubid). They found a workable approximation for the area of a circle, and thus a reasonable approximation to what we call pi.

Ancient Egyptian arithmetic was focussed on addition, with multiplication being carried out primarily through a process of repeated additions and doublings. They knew about fractions, but their attention was focussed primarily on reciprocals (in modern terms of the form {1/n} where n is an integer). Calculations which would result in other fractions (such as division or the extraction of square roots) were carried out using added sequences of these reciprocals. Tables of reciprocals were developed to assist such calculations. These insights were conveyed with the aid of simple diagrams (see the extract of the Rhind Papyrus below[78]), and mathematical ‘recipes’ rather like simple modern computer algorithms (especially in the most literal computer languages such as COBOL[79]), where the amounts of different quantities were first specified, and then the sequence of arithmetic steps that would be required to produce the required answer.[80] The geometric problems were laid out in exactly the same way, and from this point of view, appeared more as applied arithmetic than what we now think of as geometry, with its constructive proofs which derive from the ancient Greeks, of which more later. There was a flavour of modern algebra in their introduction in these of ‘aha’ - a place-holder for a quantity that was to be determined in what were from a modern perspective linear equations, and their

76. British Museum, “Rhind Mathematical Papyrus” `http://www.britishmuseum.org/explore/highlights/highlight_objects/aes/r/rhind_mathematical_papyrus.aspx` (viewed 1 Jan 2012)
77. Rossi in Robson and Stedall, *The Oxford handbook*, pp. 407–8.
78. source at 3.bp.blogspot.com/_Om5WdRNbuEE/RhR7fCaoGfI/AAAAAAAAABs/4K8uUPqpLPc/s320/rhind.jpg (viewed 30 Dec 2011)
79. see for example, `http://en.wikipedia.org/wiki/COBOL`
80. see for example, Marshall Clagett, *Ancient Egyptian Science: A Source Book*, Volume Three, “Ancient Egyptian Mathematics”, American Philosophical Society, Philadelphia, 1999.

apparent understanding that multiplication is commutative (a x b = b x a).[81] The papyrus is shown in figure 2.12 below.

Figure 2.12. Rhind Papyrus ~1650–1550 BC

The mathematics was thus of a very practical kind, with instruction being in terms always of concrete examples with specified amounts from which, presumably, the budding scribe would learn enough to then be able to do similar calculations in everyday working life. There has been found little of the emphasis on abstraction - the development of overarching proofs and theorems, abstract algebra, and the like which was certainly seen in the ancient Greek geometry and more generally mathematical reasoning. As Boyer and Merzbach put it "Even the once vaunted Egyptian geometry turns out to have been mainly a branch of applied arithmetic."[82] One reason for this lack of emphasis on abstraction may well be that the Egyptian civilisation was highly settled in its generously fertile and annually renewing Nile valley. There was not even a great need for attention to warfare, although battles did take place. But the society survived well on maintaining traditional practices and slow progression over its millenia of sustained existence. In short, it developed the mathematics it needed to meet what it saw as its challenges, and did not experience any intense incentive to develop more.[83]

This may be an explanation, but it is not necessarily the whole explanation. For example, the question arises for any society, who is doing the calculating, and for

81. For more on this see for example, Annette Imhausen, "Ancient Egyptian Mathematics: New Perspectives on Old Sources", *The Mathematical Intelligencer*, Vol. 28, No. 1, 2006, pp. 19–27.
82. Boyer and Merzbach, *History of Mathematics*, p. 19.
83. Ibid., p 20.

whom? Who is developing the methods, and why? The records we have are from scribes preparing other scribes for work in the various enterprises of the Egyptian society. This includes the work of going into the fields to estimate the annual dues, to arbitrate on behalf of the Pharonic order the disposition of land, to ensure that the work of metalurgy is carried out to meet hierarchical requirements, to prepare for religious rituals (for example the ritual measuring at the commencing of construction of a building), to estimate materials and labour required in major building projects, to account for progress, and to finally confirm the accuracy of work done.[84] But much of the actual work would have been done on the ground, with answers to practical problems, whether it was how to built to a particular slope, or how to measure a large area, being developed by practioners. In this sense, the style of this mathematics and its representation may have been because it fitted a world in which mathematics was developed in practice, with practical answers being explained to others, learned and eventually inscribed on papyrus, perhaps after much development literally "in the field". As will be discussed more later, this factor - of who the mathematics was for, and who was developing it, appears to be an important consideration in understanding the role and design in the development of new calculating technologies.

Mesopotania The Sumerian civilisation, is often described as having displayed greater innovation than that by Egypt in its mathematical development. Whilst also situated in a fertile region, part of the explanation that can be offered is that the Sumerian civilisation experienced more disrupted circumstances, and in turn displayed greater inventive vigour. Whether or not that is the whole explanation it is certainly true, that to modern eyes with the vantage point of knowing the current form and state of mathematical knowledge, the Sumerian civilisation did make some striking advances. As in Egypt, the fertile Mesopotanian valley was the home to an imposingly organised civilisation under the governance of a strong highly centralised hierarchy. In maintaining and extending the stability and authority of its rule the rulers found it necessary to mount major irrigation works designed to irrigate and control flooding from the Tigris and Euphrates rivers. However, this land was the focus of invasions from many directions including, as already mentioned, that of the Akkadians, followed by a string of successive invasions and revolts.[85] The mathematics was set within the technologies developed - the sexagessimal (base 60) number system, the cuneiform script, the use of clay as a scribal medium, the elaborate schooling system, and the calculational needs of this highly urbanised and organised society.

The successes of the Mesopotanian mathematics included the eventual development of a place holder symbol that allowed 22 to be clearly distinguished from 202 (in our script) and a realisation that fractions could be treated just like whole numbers (by similar means to our decimal numbers, albeit being written to base 60). Given this inventive direction the development of comprehensive look up tables for a variety

84. see for example, Rossi, "Mixing, Building and Feeding", *Oxford Handbook*, pp. 407–428.
85. Boyer and Merzbach *A History of Mathematics*, pp. 21–2.

of applications seems natural, and some survive.[86] Utilising tables of outcomes, together with these insights into the capabilities of the number system provided a sufficient basis for greater facility to grow in multiplication and division. From this followed the development of some remarkable capacities which included the ability to extract square roots . For example, using a simple iterative method for working out square roots of numbers, and one of the surviving tablets gives the square root of 2 (in sexagessimal as $1 + 24 \times 60^{-1} + 51 \times 60^{-2} + 10 \times 60^{-3}$) which is accurate to 5 of our decimal places. There was also a capacity to pose and answer questions which in our terms amount to not only linear, but quadratic and even cubic equations, and indeed answers to some simultaneous equations. There are even some tables of powers of numbers, which in principle enable the equivalent of logarithmic calculations to be completed. A range of remarkable other geometric and algebraic insights (see for example figure 2.13, *Plimpton 322 Tablet*, below[87]) have been recorded.[88]

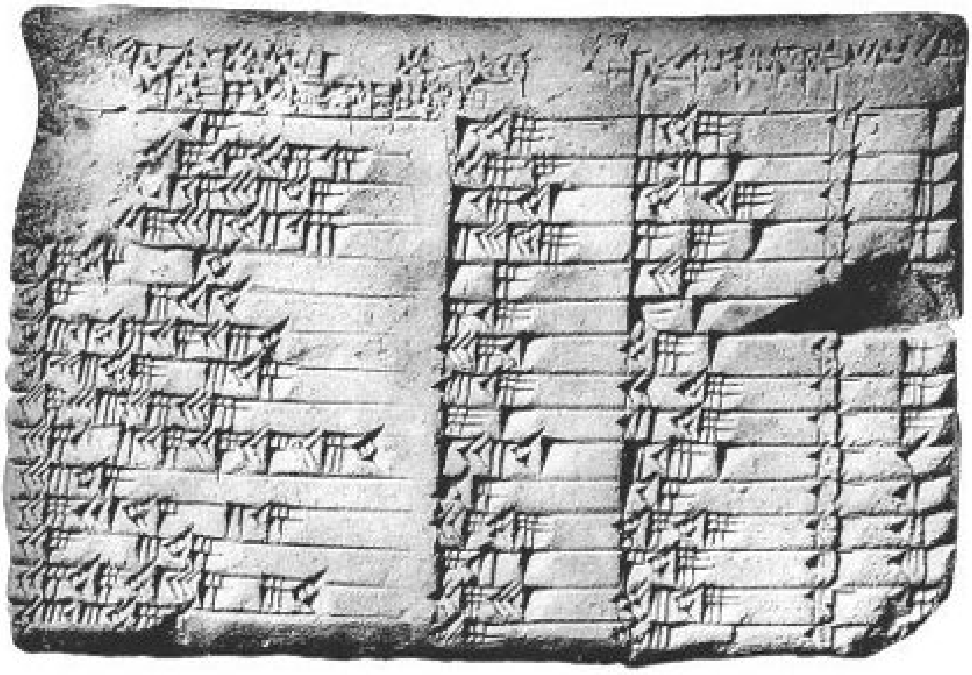


Figure 2.13. Plimpton 322 Tablet ~1800 BC (held by Yale University) - bearing what is believed to be a table of numerical relationships between the sides of triangles and associated squares. It is Old Babylonian, and its layout and syntax indicate that it is from the ancient city of Larsa.[89]

All this is very striking, yet as with the Egyptians there is no evidence that the Sumerian civilisation or its later manifestations saw any need to prove their methods. Nor do we have much evidence of what role counting devices played in creating these

86. Ibid p. 25
87. From Boyer and Merzbach *A History of Mathematics*, p. 34.
88. Ibid pp. 26–39

numerical rules, even though the abacus itself is believed to have been introduced into the ancient Sumerian society.

China and India China and India were also sites of ancient civilisations which as with Egypt, probably drew on developments in Sumeria but also were sufficiently isolated and long-standing to develop their own significant bodies of mathematical work. As with the Sumerian and Egyptian mathematics these were developed as solutions to practical problems, which were then elaborated also into solutions of teasing questions. Built on prior solutions to previous questions, new questions could then be developed. The answers however, were usually in the form of mathematical 'recipes'.

In both cases the surviving historical evidence is not sufficiently robust to confidently answer the question of how far back this work extended. There is reason to place the first Chinese empire as stretching back to 2750 BC, although more conservative estimates assert a closer date of around 1000 BC. There is no more agreement on the dating of the oldest chinese mathematical work, the *Chou Pei* with estimates ranging from 1200 BC to 100 AD. Boyer in his imposing work opts for 300 BC.[90] The *Chou Pei* and subsequent works reveal a mixture of "accurate and inaccurate, primitive and sophisticated results".[91] Certainly the Chinese mathematical development appears to have largely been of Chinese origin, although early lessons may well have be drawn from interchange with Mesopotania. The areas of triangles, rectangles and trapezoids are correctly calculated in the context of problems, and the area of the circle (with pi approximated by 3). Magic squares, the solutions to problems which would now be considered simultaneous linear equations, and the like are also solved. The development was not continuous, with developments disrupted by the occasional burning of books and other interruptions. By the fifth century Tsu Ch'ung-chih (450–501 AD) had performed the notable feat of establishing the value of pi to 6 decimal places. However, there remained sparse availability of written works even though printing was developed in China as early as the eleventh century AD.[92] Two important treatises from 1299 AD by the great mathematician Chu Shih-chieh (1280–1303 AD) marked a peak in the development of Chinese algebra and revealed an understanding of how to approximately solve some quite sophisticated problems (for example, in modern terms, ones involving variables up to the 14th power - ie. x^{14}).[93]

Similarly in India there appears to have been an old but sophisticated civilisation contemporaneous with the time of the construction of the Egyptian pyramids. Once more we see references to early arithmetic and geometric insights going back to 2000 BC, but there are no surviving documents to confirm this. Boyer considers it likely that India also had its "rope stretchers" to assist in the construction of temples and the like perhaps contemporaneous with the founding of the Roman Empire (from 753 BC). But any dating of this is speculative. Hindu mathematics has even greater

90. Boyer and Merzbach *A History of Mathematics*, pp. 195–7.
91. ibid p. 196
92. ibid p. 198–203
93. ibid p. 204

discontinuity than that in China. The first known Indian mathematical text (by Aryabhata) is placed much later, traditionally about 476 AD (the time of the fall of the Western Roman Empire). There is subsequent evidence of significant geometric and algebraic insights, the calculation of pi (but less accurately than in China) and the development of astronomical measurements, all of which shows some influence from Greek mathematics (described below).[94]

As for improvements in calculation, the key Hindu invention of a system of numerals in which successive places stood for powers of ten has already been mentioned. This is explained in the text by the Hindu mathematician Aryabhata who in the sixth century noted that he carried out his calculations using a notation where "from place to place each is ten times the preceeding". However, the earliest known inclusion of the numeral for zero (to represent an empty space) is from 876 AD.[95] The Chinese are notable in this regard for their use of rods (black for things being added, and red representing things being taken away) and their subsequent representation of this with an equivalent set of numerals, the whole being controlled by use of horizontal rods to represent multiples of ten.

From what is available it does seem that the questions, and their answers, in all the civilisations mentioned above - Sumerian, Egyptian, Indian, and Chinese - were largely formed around practical problems (such as determining the dimensions and areas of regular bodies, and in particular triangles). In the course of this Pythagorean relationships between the lengths of sides were tabulated. These questions, intermediate answers, and methods for using these to give final answers were clearly systematically developed. The working of examples was greatly facilitated by developing appropriate scripts for writing that down. Technologies such as counting rods, sand trays, and later the abacus were used to complement that work. All of this was of practical importance, and at times also celebrated in high places. Significant texts appeared by leading mathematical scholars which summarised and taught what had been achieved. But none of this produced a body of mathematical knowledge in the systematic and abstract way that is both required and celebrated in modern mathematics. That is, the knowledge was not founded on methods of proof to establish its authority. The place where the desire, and methodology to do this emerged was in the mathematical writings of an elite group of mathematical philosophers in ancient Greece.

Greece - philosopher and practitioner - a "two track" mathematics.

There is more than one way of establishing the authority of a claimed truth. Religious pronouncements may base their claim to truth on the authority of a deity perhaps revealed through some particular humans. Then there is the authority of repeated observation. Repeated observation are the basis of common sense: that the sun will rise every day, that things released will fall, or that water heated enough will turn into

94. ibid pp. 206–11.
95. ibid p. 211–3

steam. These observations may over time become so well accepted that they may be accepted as general laws.

Another form of authority, is that used to support the claims of truth in modern mathematics and the sciences. This is the authority which derives from clearly laid out logical deduction. Logical reasoning of course is used in all forms of argument. But the key to the way it is used in mathematics and science is that it is written down according to certain conventions. These allow the various steps of the argument to be seen very clearly, and if each is accepted as following logically from the previous, and the starting point is already accepted as being true, then the claim that the whole is true can be particularly convincing.

This highlights the difference between what surviving mathematical records reveal was done in ancient Egypt and Mesopotani (based on "recipes" for solving specific types of problems, usually based on an example), and the more abstract mathematical reasoning that emerged from ancient Greece and gave rise to our modern concepts of "proof".

It is not necessary here to dwell for long on the expansive achievements of the ancient Greeks in the development of abstract geometry and mathematical reasoning. All this is described both in great detail, but also accessibly in summary elsewhere.[96]

There is a shadowy history of mathematics in ancient Greece. It begins with invaders from the north in the second millennium BC. They had no known capacities in literacy or numeracy. After that was the likelihood of trade and other interchanges with Egypt and Mesopotania. Later we have the sophisticated Greek literature, already evident by the first Olympic Games in 776 BC, and then the beginnings of the formal abstract mathematics for which the ancient Greeks have been so celebrated. That may be traced back to the illusive figures of Thales of Miletus (around 585 BC) and his use of geometry to solve practical problems, and Pythagoras of Samos (around 580–500 BC).[97]

The development of mathematics from these sources (and the oral mathematical knowledge which they may have formalised) flowered until the destruction of the Academy of Athens in 529 AD. That period saw the emergence of "schools" of mathematical philosophers including the magisterial Pythagorean School with its emphasis on proof (including, so it is believed, the proof of "Pythagoras's Theorem"), Plato's Academy in Athens (which became a centre of mathematics in the 4th century BC), and associated achievements including an iterative method used to determine the areas and volumes of complex curved and other objects. The achievements were famously brought together by Euclid in his Elements in the 3rd century BC in which the formalisation of what we now understand as 'mathematical' rigour and its use for "proof" was systematically displayed. Thereafter the "Golden Age" of Greek

96. see for example `http://en.wikipedia.org/wiki/History_of_mathematics` (viewed 14 Jan 2012) and references contained therein.
97. Carl B. Boyer and Uta C. Merzbach, *History of Mathematics*, Wiley, 2010, pp. 43–52.

mathematics began to decline although there was nevertheless a series of significant analytic developments, especially in algebra.

It is not clear that there was any significant advance in Greek mathematics over the three hundred years from 150 BC to 150 AD. By then it was clear that the period of rapid growth of this field in Greece was at an end. It has been suggested that this decline was caused by an emerging emphasis on practical application. Others attribute the decline to difficulties now encountered in the approaches adopted. Others again attribute it to the waning power of Greece in relation to the military might of Rome.

There was a period from 250 to 450 AD when some innovative mathematicians made continuing contributions, notably Nichomachus of Gerasa (~100 AD) who wrote *Introductio arithmeticae*, Diophantus of Alexandria ~250 AD the author of a thirteen book treatise *Arithmetica*, and Pappus of Alexandria (~320 AD) who wrote his important *Collection (Synagoge)* of Greek mathematics. These various contributions are well described elsewhere.[98] The mathematical outputs from Alexandria, which had become the centre of Greek mathematics at the time of Euklid (~300 AD), after a further 100 years, had come to an end.[99] Over the following century, some further development occurred, for example, through Proclus of Alexandria (410–485 AD) who went to Athens and wrote an important *Commentary on Book I of the Elements of Euklid.* However, by ~500 AD not only Greek, but also as will be described, Roman mathematical development (and with them the entire production of systematic abstract mathematical development from the ancient world) had ceased.

The above is related to provide part of the context that of the development of calculating technology. Importantly, the abstract, and thereafter much celebrated invention of formally and systematically written abstract mathematical (and other applications) of reasoning, does not represent the only mathematics that was being performed. Indeed there was at least what Marcus Asper has styled "the two cultures of mathematics" in ancient Greece.[100] To be more precise Asper stresses that recently "a consensus has emerged that Greek mathematics was heterogenous and that the famous mathematicians are only the tip of an iceberg that must have consisted of several coexisting and partly overlapping fields of mathematical practices."[101]

This picture of a tapestry of mathematical practices being utilised by different participants in the society must be closer to the reality than two totally separated cultures. (The old joke comes to mind that 'there are two classes of people: those who divide the world into two classes of people, and those who don't'). Nevertheless, with that caveat it is useful to reflect on the fact that *at least* two practices were in play: the abstractions being developed by a relatively small philosophically inclined elite (what Asper calls "theoretical mathematics"), and the continuation of the practical mathematics in the style of useful recipes for practical purposes in everyday activities,

98. Boyer and Merzbach, *History of Mathematics*, pp. 176–191
99. ibid, p. 192
100. Marcus Asper, "The two cultures of mathematics in ancient Greece", ", in Robson and Stedall, The Oxford handbook, p.107.
101. ibid

which was the legacy of Mesopotania and Egypt, to name just two major centres (which will be referred to here as "artisanal" or "pragmatic" mathematics and which Asper refers to as "practical mathematics"[102]).

As Lloyd points out,[103] this distinction between the abstract and pragmatically applied, can be found in Plato's remarks on the quite different types of usefulness embodied in, on the one hand, meeting the needs of everyday life, and on the other of training the intellect. But the search for proof is the most striking feature of the abstract work of the Greek philosopher-mathematicians. And here we may see how the political culture of the time may have sharpened the desire for this. For the world of the Athenian free citizens was governed through the law courts and assemblies. In these, as Plato stresses, mere rhetorical skill could be sufficient to sway the participants, whatever the actual truth. But the claim that could be made for mathematical conclusions was that they were exact and proven. More generally, philosophy sought the same strength of truth and philosophers celebrated, developed, and recorded the types of mathematics which could be shown to meet this rigorous standard.

Whilst there are only glimpses of the more pragmatic mathematics we get a taste of it in Aristophanes play *The Wasps* from 422 BC:

> BDELYCLEON: "Listen to me, dear little father, unruffle that frowning brow and reckon, you can do so without trouble, not with pebbles, but on your fingers, what is the sum-total of the tribute paid by the allied towns; besides this we have the direct imposts, a mass of percentage dues, the fees of the courts of justice, the produce from the mines, the markets, the harbours, the public lands and the confiscations. All these together amount to nearly two thousand talents. Take from this sum the annual pay of the dicasts;[104] they number six thousand, and there have never been more in this town; so therefore it is one hundred and fifty talents that come to you."
>
> PHILOCLEON: "What! our pay is not even a tithe [one tenth] of the state revenue?"[105]

Even in these couple of lines we seen enough to establish the gulf between the practical problems to be dealt with in day to day life, and the sophisticated highly abstract geometric reasoning of the classical Greek mathematicians. On the one hand, for example, is the highly abstract question of how to develop a pure geometrical method or system of harmonic ratios in order to understand musical harmony, or to prove a theorem about the volume of a geometric object in terms of its sides. This was the sort of question the tackling of which could be seen as a good intellectual

102. ibid pp.108–114
103. Lloyd "What was mathematics in the ancient world?", *The Oxford handbook*, pp. 9–10.
104. Greek citizens chosen each year in ancient Athens to sit in judgement on issues brought before them
105. `http://classics.mit.edu/Aristophanes/wasps.html` *The Wasps* by Aristophanes, English translation provided by The Internet Classics Archive (viewed 19 Jan 2012).

preparation for the more important art of philosophical reasoning wherein the 'really big questions' could be tackled.[106] On the other hand there was the practical question of how to keep track of resources from multiple sources. The methods for tackling this included the use of calculational aids (of which pebbles as counters were a frequently used technology). The former development in abstract mathematics was characteristically Greek. The latter, was common to all mathematically literate and complex societies (including those of ancient China, Mesopotania, Egypt and Greece).

What can we make of this? Asper argues that the abstract or theoretical "culture" of mathematics was developed by a small group of elite Greek citizens with an impetus that was partly aesthetic, partly a 'game' in which successful Athenian 'gentlemen of means' who could afford such pursuits would compete without the need to gain financial return.[107] In place of wealth they gained prestige (and no doubt associated authority. This elite practice was focussed on communicating through a developing orthodox written form of discourse, general theorems about ideal geometric forms. The work emerged in the sixth to fifth century from practical roots. But the thrust was sharply against the 'vulgarity' of meeting practical needs. Indeed in its mode of presentation and areas of work it was directed to maintaining a sharp distinction between crude application versus the intellectual abstract search for proved knowledge which lay at its core.[108]

The less celebrated and less visible practical mathematics was more closely aligned to its anticedents in Egypt and Mesopotania, based around practical recipes for solving the multiple problems of daily life in a substantial and sophisticated urban society. Those problems were associated with the commercial, engineering, agricultural, religious, political and administrative challenges encountered in daily life. As Asper notes, this mathematics was derived from older traditions from the Near East, focussed on 'real-life'problems, communicated actual procedures which by example illustrated more general approaches to encountered problems, and relied on written texts only in a secondary way if at all. Oral communication and guild training were the means of passing on the relevant approaches to practitioners in particular fields of work.[109]

Thus in the Greek story, the evolution of calculation merges into an evolution of mathematics, but divided across a spectrum marked by the pragmatic and practical at one end, and the theoretical and abstract at the other. Spread across that also are different forms of possessing and transmitting knowledge - from those of the philosophically oriented schools of abstract mathematics and their formalised presentations of written proofs (in a dispassionate and subjectless style still characteristic of modern scientific communication) to the specific oral and apprenticeship styles of transmission utilised by the practical problem-solving artisans.

106. ibid, pp. 9–18
107. Asper, "The two cultures of mathematics in ancient Greece", p. 124.
108. ibid, pp. 128–9.
109. ibid

The geometry of forms becomes the paradigm physical embodiment of the abstract end, whilst numbers, and simple calculational devices for manipulating them, most clearly the use of “pebbles” (whose origins are already shrouded at the dawn of humanity) is situated at the other. Pebble arithmetic, together with the means of measuring, weighing, sighting angles, and the like never appears in the theoretical accounts. But it has left sufficient shadow through sporadic references (such as in Aristophanes quoted earlier) to indicate that the use of pebbles as counters, often on a marked board, was widely used.[110]

Of course, quick use of pebbles on a special board is not an innate skill. Almost certainly its use was commissioned and practiced by a guild of skilled practitioners and many tricks could be developed to speed the process up, just as with the abacus in Modern Asian societies. The manipulation of pebbles, in patterns, and then stacked into volumes, is enough to provide a wide range of mathematical insights. So whilst the written works of the abstract mathematical game playing elite may never have mentioned it, an unknown number of insights no doubt translated from the more practical pursuits of the pragmatic world of calculation, with its pebble counting technologies and the associated group of professional pebble counting practitioners.[111] In this sense the difference between the two tracks or cultures of mathematics was not just one of goals, nor of abstract versus pragmatic, but also of social class. An analogous social differentiation framed the use of calculating technology in ancient Rome.

Roman parallels

The Roman Republic (510–44 BC) gave rise to the Roman Empire which at its height stretched from England to the Persian Gulf, and ended with the fall of its Western half in 476 AD. Boethias (CE ~480–524) was perhaps the foremost mathematician produced by ancient Rome who also wrote a work on ethics, *De consolatione philosophae*, as he faced execution having fallen out of favour with the Emperor.[112] His death effectively coincided with the end of mathematical development within the ancient Roman empire.

In fact, whilst the Roman society was a powerful military and organised system, its elite was not much attracted to the power of mathematical investigation, contributed little to what is known about mathematics, and gained much of its practical knowledge of mathematics from the civilisations Rome conquered and with which it traded, not the least from the ancient Greeks.[113] The development of Roman numerals, and the importance of the use of the abacus in ancient Rome in manipulating them efficiently has already been discussed. Beyond this only one further aspect of the use of mathematics in Rome will be considered here as a useful illustration of a more general conclusion.

110. ibid pp. 108–9
111. ibid p. 109.
112. Boyer and Merzbach, *History of Mathematics*, pp. 191–192
113. Boyer and Merzbach, *History of Mathematics*, p. 177

For even the limited synoptic history described already the relationship between the emergence of these mathematical systems and the needs and organisation of the societies is quite striking. In each case the evolution of counting and the technologies to facilitate that, has been shaped by, and helped shape the types of society which could be constructed. For the Egyptian and Sumerian civilisations which could settle in relatively climatically stable and fertile valleys, the consequent emergence of agricultural practices, and the correspondingly more settled agricultural societies led to the possibility and growth of a more complex urbanised social organisation. Work could emerge as a differentiated set of skilled and less skilled roles, with more diversified activities, including trade and barter. The regulation and arbitration of disputes related to this sort of activity developed along with some form of broader organisation of ruler and ruled, with accompanying trappings of allocation of land and other resources. Thus, the Old Babylonian laws (detailed in cuneiform tablets) laid down by King Hammurabi in the *Code of Hammurabi* from 1792–1750 BC lay down some 282 laws dealing with everything from the treatment of theft and adultery to the duties of workers, property rights, prices for services, obligations to neighbours,the graduated punishments" associated with "an eye for an eye", the precept of "innocent until proved guilty" and an ancient form of minimum wage.[114]

Associated with the development of these more complex social orders there was the potential for the growth of powerful elites who could both deploy public resources for collective purposes (such as defence, irrigation and religious ritual), impose corresponding forms of taxation and record a census of the population and the extent to which citizens met their tax and other obligations. These increasingly complex forms of social organisation could lead to the desire and need for recording of roles, debts and assets, and more complex construction projects requiring estimation, measurement, and design.

Different social structures also support particular forms of calculation. Thus even with the legendarily cumbersome form of the Roman numerals, the society was now structured in a way in which a wealthy set of merchants and high officials began to rely on calculational devices and those who could operate them. It was not hard to access necessary labour. The picture below, figure 2.14,[115] from a gravestone in the Museo Capitolino in Rome, shows a first century AD Roman merchant with his "calculator"[116] - the person on the left who is using a hand abacus to tally amounts at the "dictation" of his master. A scribe to the right takes down the results.

There is a subtle issue to be careful with in the above argument. It is generally an oversimplification to suggest that a social change led to a technological change or vice-versa. After all, it is certainly true that more complex forms of social organisation created the impetus to develop better forms of calculation, but it is also true that the invention of more sophisticated forms of calculation allowed these more complex

114. *Hammurabi's Code of Laws*, tr. L. W. King, `http://www.sacred-texts.com/ane/ham/ham04.htm`, viewed 23 March 2014.
115. Menninger, *A Cultural History of Numbers*, Fig 183, p. 306.
116. cf. Latin noun: calculator - arithmetician or accountant; verb: calculo, calculare, calculavi, calculatus sum - reckon or calculate

Figure 2.14. Roman calculator(s) at work. ~0–100 AD

forms of organisation to develop and flourish. What we can say is that the two - more complex society (in the sense of greater interdepenency and interaction across larger numbers of people and institutions), and corresponding governance systems (of a variety of types) - co-evolved with more sophisticated forms of mathematical capacity (which, as argued elsewhere, is part of a broader capacity for reflexivity).[117]

In particular, similarly to their Greek counterparts, the Roman capacity to meet the need to calculate in a complex hierarchical society was greatly facilitated by the use of counting boards, and in a more mature form, the abacus. As with the Greeks, it was not necessarily the user of the calculations who performed them. That task could be left to subordinates (whether free or slaves) who had gained the necessary skills by studying under a master of the art. Thus the abacus, as a calculational technology, together with the calculator who was skilled to use it, formed a symbiotic pair easing the processes of commerce, administration and engineering, in the developing Roman society.

Measuring, timing, calculating and astronomical prediction.

Whilst this discussion is focussed on calculation, the need for this is of course only one aspect of the developing needs of a complex society. As already mentioned, measurement has been equally crucial, and measurement and calculation form parts

117. This argument is made in extensive detail in `http://worlds-in-transition.com` by Joseph Camilleri and Jim Falk, *Worlds in Transition: Evolving Governance Across a Stressed Planet*, Edward Elgar, London, UK, 2009.

of a bigger whole. In all the evolving social settings mentioned so far, measurement has played an important role. This ranges from the work of the rope stretchers of Egypt (and Athens) to the early methodologies for measuring time (for example with sand and water flows,[118] and the burning of graduated candles) in ancient Rome.

It is not surprising that the activities of the motions of the lights in the sky as they appeared in day and night would be the focus of much interest in many societies. The mysterious motions of celestial objects, the unatainable remoteness of the celestial sphere, clear influence of its activities with the seasons, and apparent correlations between celestial motions and weather, tides, lightening and thunder, and occasional destructive impacts on the earth pretty much guaranteed that these would be the subject of speculation and mystical interest. Many would claim special knowledge of these motions and their implications, ranging from the mystical claims of priests to those of astrologers. Their predictions and interpretations could be highly influential. Evidence of this can be found right through the archaeological record.

There was thus in most societies a relationship between religion, astrology and astronomical observation. In more sophisticated cultures this included also astronomical measurement. Many ancient structures and devices can be identified which served to measure astronomical events. They were used to predict future movements of the sun, moon, planets and stars, the seasons, and religiously significant events shaped by these. Many such devices have been found ranging from possible astronomical implications of the sarsen circular stone monument at Stonehenge[119] erected in about 2200 BC to devices explicitly constructed to measure the angle subtended above the horizon by a star.

Astrology in particular is a central theme, based of course around different conventions for describing celestial configurations and events and different interpretations. Given no better explanation it was a reasonable presumption that the movements in the sky followed the dictates of gods, and thus could be a guide to their intentions for the world of humans. Thus the attempt to utilise these to make predictions in human affairs can be found in communities ranging from those of ancient Babylonia through even to Modern societies. Evidence of astrological considerations can be found in ancient Egyptian, Greek and Roman temples. Corresponding beliefs and practices in magic, religion, philosophy, mathematics, and astronomical observation can be found mixed together in a variety of forms. Numerous astrological works can be found written in Greek and Latin (generally incorporating earlier Egyptian and other astrological concepts). The earliest surviving systematic treatise was

118. For a description of the Roman understanding of the celestial movements, the implications for the hours in a day, and the construction of sundials and water clocks to measure the passage of time in relation to these movements, see Marcus Vitruvius Pollo (~ 80 BC–15 BC), *de architectura*, translated by Morris Hicky Morgan as *The Ten Books of Architecture*, Book IX, `http://en.wikisource.org/wiki/Ten_Books_on_Architecture/Book_IX` (viewed 25 Jan 2012).

119. For a subtle discussion of the role and design of Stonehenge see for example Lionel Sims, "The 'Solarization' of the Moon: Manipulated Knowledge at Stonehenge", *Cambridge Archaeological Journal*, vol 16, 2006, pp. 191–207

Manilius's *Astronomica* written in Latin verse in the the first century (C1 AD).[120] Early astrological instruments not surprisingly included variants of technologies used in other fields, from the counting board to the use of diagrams, inscriptions and calculations on papyrus, clay, stone and cloth. Using these, horoscopes and other astrological information could be developed and communicated. In particular, astrologers developed boards on which different configurations of the planets and stars could be illustrated by means of "pebbles" or more stylised counters. These counters could be moved to show the relationships of planets to the signs of the zodiac, to each other, and to the horizon. A surviving example, figure 2.15 is shown below.[121]

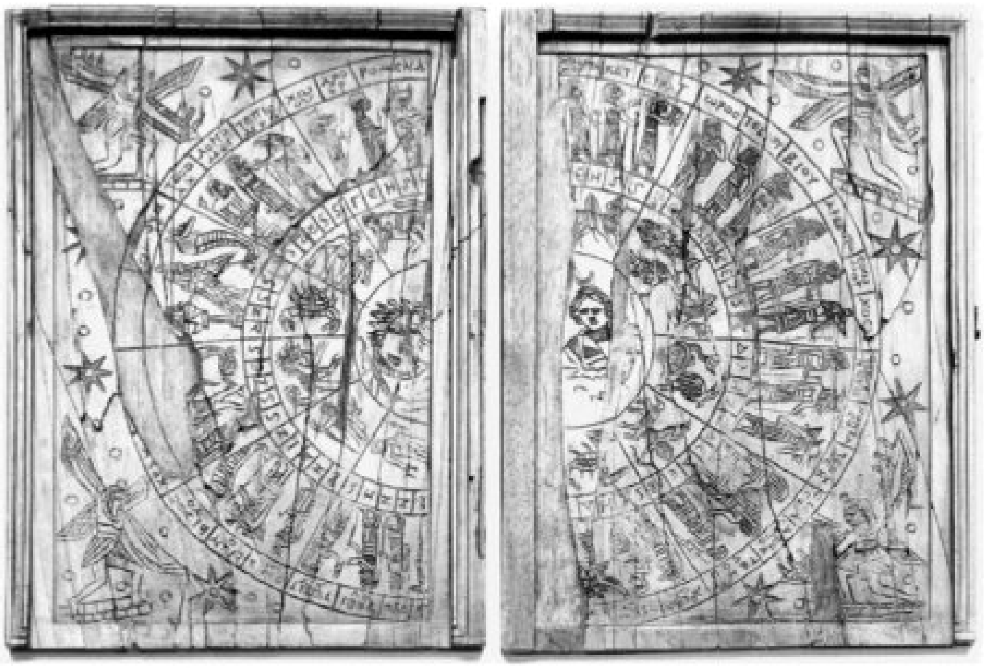


Figure 2.15. Ancient Greek astrological board - made of ivory, discovered at Grand (Vosges) C2 AD. From the outside, the concentric rings show names of ancient Greek astrological divisions (Decans), corresponding figures, terms expressed in Greek numerals, the Zodiac, and busts of Helios and Selene.

By the time of the first century AD increasingly methodical observations of the sun, moon, known planets, and background stars had been undertaken. It was with this heritage that Claudius Ptolemy (90 AD - 168 AD) set about writing his magisterial work on astronomy. It achieved in that field similar standing to that achieved by Euklid in his *Elements* in relation to geometry. Ptolemy was a Roman citizen, writing in Greek, and living and working in Alexandria, which was by then the capital city of the Roman province of Egypt. By now he had at his disposal not only the full power of Greek mathematics now at its zenith, but also Arisotle's philosophically constructed picture of the earth. This had the Earth as the stationary centre of a cosmos

120. James E Evans, "The Astrologer's Apparatus: A Picture of Professional Practice in Greco-Roman Egypt", *Journal for the History of Astronomy*, Vol 35, Part 1, No 118, February 2004, pp. 1–44.
121. ibid, Fig 1., p. 6.

around which the stars and planets moved in circles. Ptolemy also had available a history of recorded Greek astronomical observation (probably all of which works were available through the Library of Alexandria) - including specific references to Meton of Athens (C5 BC), Callipus of Cyzicus (C4 BC), Aristarchus of Samos (C4–3 BC), Eratosthenes of Cyrene (C3–2 BC), and most notably Hipparchus of Nicaea (162–127 BC).

Titled variously as what transliterates as *Mathematike Syntaxis* in Greek, *Syntaxis Mathematica* in Latin, or *The Almagest* (the "Greatest") by arab translators (which became the English title) and then printed in England (in Latin) as *Almagestum* (in 1515 AD),[122] the book shows how with the Earth, as a stationary frame of reference, it is possible to build a model of the motion of the solar system as seen from the Earth. In this the Sun moves daily in a circle around the Earth, and each planet moves in a combination of two circles. The major circle (the deferent) is slightly offset from the Earth (by half the distance from a point for each planet known as the Equant), and the center of the other minor circle (the epicycle) moves around the circumference of the major circle. The model was remarkably accurate in relation to the observed motions of the planets. It provided a clear explanation of why the planets have apparently "wandering" paths in relation to the fixed stars. Ptolemy discovered that offsetting the centres of the deferents by the use of the Equant gave a greatly improved correspondence between theory and observation.[123] So good was the result that Ptolemy's model was to serve for some 1400 years until displaced, after a long ideological struggle by the heliocentric theory of Nicolaus Copernicus (1473–1543),.[124]

The Almagest was thus the most authoratitive exposition of astronomy to be produced in the pre-modern period. In it, as well as laying out the theory of the motions of the celestial bodies, Ptolemy lists the key measuring devices used to observe them. For example, he refers to the gnomon (a vertical stick from which the shadow of the sun as it moves can be measured) and armillary spheres (an astronomical device showing the concentric rings representing the major circles of the celestial sphere),[125] and a mural quadrant (a graduated quarter circle inscribed on a wall, by means of which the elevation of celestial bodies could be measured).[126] The picture figure 2.16 below

122. Claudius Ptolemaeus, *Almagestum: Opus ingens ac nobile omnes Celorum motus continens. Felicibus Astris eat in lucem*, 1515. Copy in the Institut für Astronomie, Universität Wien, Türkenschanzstraße 17, 1180 Wien reproduced at `http://www.univie.ac.at/hwastro/books/1515_ptole_BWLow.pdf` (viewed 31 Jan 2012)

123. Richard Fitzpatrick, A Modern Almagest: An Updated Version of Ptolemy's Model of the Solar System, University of Texas at Austin, `http://farside.ph.utexas.edu/syntaxis/Almagest.pdf`, (viewed 27 Jan 2012), p. 6.

124. Nicolaus Copernicus, *De revolutionibus orbium coelestium*, Holy Roman Empire of the German Nation, Nuremberg, 1543

125. Ptolemy, Almagest, Book 1 section 6 or . For a translation see `http://www.brycecorkins.com/wp-content/uploads/documents/PtolemysAlmagest.pdf` (viewed 28 Jan 2012)

126. William Cecil Dampier, *A history of science and its relations with philosophy & religion*, University Press, Cambridge, 1929, p. 49.

is a representation from 1564 of Ptolemy with a quadrant and armillary sphere.[127] The armillary sphere, carried here by a Satyr in a statue from 1575 can be seen in figure 2.17 immediately following.[128]

Figure 2.16. 1564 AD: Ptolemy with quadrant and armillary sphere

Figure 2.17. 1575 AD: Ptolemeic armillary sphere carried by satyr

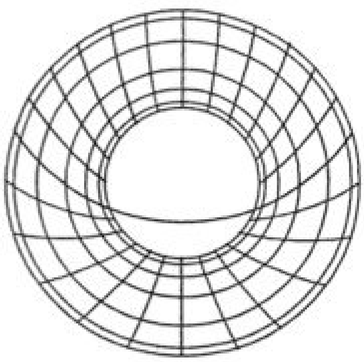
A more refined device, the astrolabe, which combined the use of a quadrant with a form of mapping the stars through stereographic projection of their paths onto a

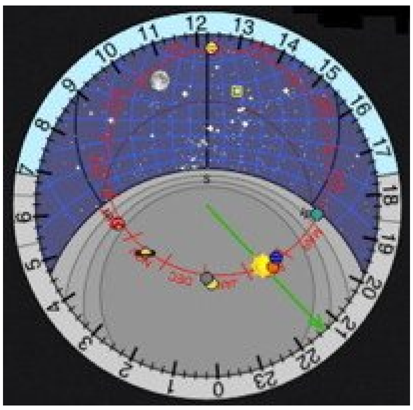
127. museo galileo Institute and Museum of the History of Science, *VII.36 Model of the solar orb* `http://brunelleschi.imss.fi.it/museum/esim.asp?c=407036` (viewed 28 Jan 2012)
128. annon, Ptolemy from *Clavdio Tolomeo Principe De Gli Astrologi, et De Geografi*, Giordano Ziletti, Venezia, 1564 `http://www.er.uqam.ca/nobel/r14310/Ptolemy/Ziletti.html` (viewed 28 Jan 2012)

plane, is attributed by some to Hipparchus (162–127 BC) who formalised the method of projection which was later utilised in the evolving device.

Perhaps the earliest description of an astrolabe like device appears once more in Vitruvius's *De architectura* (~88–26 AD) where he describes an anaphoric clock.[129] This device consisted of a large vertical disc rotated by a water wheel or other mechanism so that it turned through a complete revolution from sunrise of one day to the sunrise of the next. On the disk are marked the stars (or constellations of stars) of the Northern Hemisphere (together with those south of the equator down to the tropic of Capricorn - whose projection forms the outer limit of the disk). A circle represents the Zodiac - the path taken by the Sun in its journey across the sky over the year. Along the Zodiac circle are 365 holes, one for each day. On each successive day a marker for the Sun is to be advanced one hole to take account of the changing lengths of the day with season. In front of this rotating disk is a fixed disk of wires. A vertical wire marks the meridian (which divides the Earth from East to West). Concentric circles mark out selected months. Radial curved wires represent the 24 hours of the day as the Sun on its disk rotates behind them. Made for a particular location, a further curved wire arc lays out the horizon (below which stars on the disc still rotate, but cannot be seen at that place). This device, see table 2.3, *Anaphoric grid and modern virtual astrolabe*, below, thus displays the positions of selected groups of stars in the sky as they appear to move from a fixed point on Earth with the passing hours.[130]

Table 2.3. Anaphoric grid and modern virtual astrolabe





Left: a schematic representation of the wire grid pattern placed in front of anaphoric clock disk. Concentric circles mark lattitudes, radial curves mark the hours, wide arc marks the horizon[131] Right: is from a modern representation (from an "ipad app") of

129. Vitruvius, *De architectura*, Book VIII, "Sundials and Water Clocks", sections 8–15^ `http://www.mlahanas.de/Greeks/Texts/Vitruvius/Book9.html` (viewed 29 Jan 2012)
130. A. G. Drachmann, "The Plane Astrolabe and the Anaphoric Clock", *Centaurus*, vol 3, issue 1, 1970, pp. 184–9
131. Drachmann, "The Plane Astrolobe", Fig. 1, p. 184.

an "astrolabe clock"[132] for a location on Earth 38 degrees North and 75 degrees West on 12 February 2012 at 9.22 PM. Sun, moon and planets can be seen on the Zodiac (red circle). Below the horizon is shaded gray. Green arrow through the sun shows the time on the 24 hour clock on the perimeter. In this modern representation the blue grid marks the angles of objects in vertical (altitude) and horizontal (azimuth) planes.[133]

The astrolabe itself is based on a similar idea but instead of the clock has a scale from which time can be read if the projected paths of the stars and hour is aligned to match the observed position of a selected star or stars. It was described by the Greek and mathematician scholar Theon of Alexandria (335–405 AD). (Theon was the father of daughter Hypatia (350–415 AD), whose unpleasant death at the hands of a Christian mob effectively ended the age of mathematical development in Alexandria.[134]) Theon's description later would become a frequent reference for rediscovery of the device in later eras.

Below (table 2.4, *Anaphoric clock and fragment*), is shown a modern representation of an anaphoric clock constructed by Prof . Kostas Kotsanas and his students. Next to it is an illustration from the 1886 Hoffmann's catalogue for the sale of one of two fragments of such clocks discovered in the 19th century, in this case at Grand (Vosges) France. The sun marker is shown set at the second hour on the seasonally varying hour lines.

A photo of the front of an early astrolabe made by Jean Fusoris of Giraumont in the Ardennes region of France (1365–1415) and now held in the Adler Astronomy Planetarium and Museum in Chicago[139] is shown in figure 2.18 below.

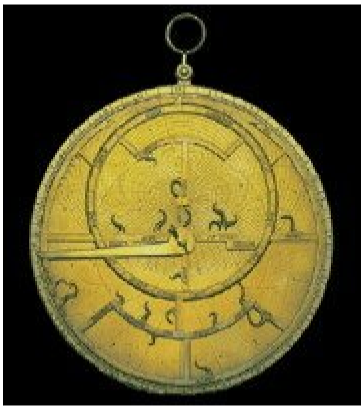
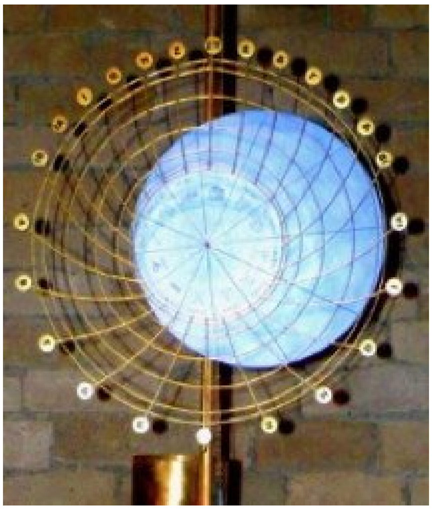
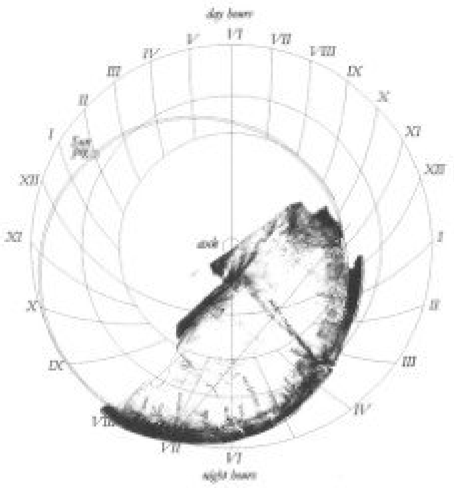


Figure 2.18. Early Astrolabe ~1400 AD by Fusoris 1365–1414

132. For a nice description of the progressive development of the astrolabe clock see George Burnett-Stuart, "Astronomical Clocks of the Middle Ages: A Guided Tour", `http://www.almagest.co.uk/middle/astclk.htm` (viewed 2 Feb 2012)
133. `http://itunes.apple.com/au/app/astrolabe-clock/id421777015?mt=8` (viewed 1 Feb 2012)
134. Boyer and Merzbach, *History of Mathematics*, p. 192
139. Front of Fusoris astrolabe (Photo from the Adler Planetarium and Astronomy Museum, reproduced at `http://astrolabes.org/pages/fusoris.htm` (viewed 29 Jan 2012)

Table 2.4. Anaphoric clock and fragment

	
Representation of an anaphoric clock	Anaphoric fragment ~C2 AD
Recent replica[137]	Catalogue illustration 1886[138]

As the above suggests, the confluence of Greek geometric developments, philosophic inclinations, and systematic observation (albeit with the aid of what to Modern eyes appear comparatively simple instruments) was able to give rise to a mathematical description of the cosmos that was to serve for more than a millenium. The armillary was a simple static representation of some of these basic ideas. The anaphoric clock and astrolabe used more sophisticated geometric ideas to provide a more practically useable representation of the changing movements of constellations but with careful settings having to be made for day and either time or position of stars. However, extraordinarily, it appears that contemporaneously with this work, a mechanical mechanism had been devised which could model movements of the stars through time in a much more fluid and autonomous way enabling important and highly valued astronomical predictions to be made. This device, the most elaborate known to have existed in antiquity (and the most reminiscent of a Modern mechanical calculator or clock), is the Antikythera mechanism.

The Antikythera mechanism A heavily corroded and encrusted set of 82 remnant fragments was discovered by Greek sponge divers in 1901. The fragments lay embedded amongst a range of treasures from C2-C1 BC in the wreck of an ancient Roman galleon at a depth of 60 metres off Point Glyphadia on the Greek island Antikythera. Whilst there has been more than a century of speculation on its mechanism and meaning the fragments have now yielded much greater detail to modern imaging technology. This reveals a highly complex ancient Greek mechanism complete with inscriptions which have been translated. In 2008 the mechanism was identified in an article in *Nature* as consisting of a device with a bronze system of interlocking cogs, and front and back output dials. The device, originally housed in a wooden frame, had some 30 intermeshed cogs which represented calendar cycles[140] and the accuracy with which the mechanism was constructed is considered greater than any later known devices until clockwork mechanisms developed in the Middle Ages a thousand years later.[141] Major fragments are shown (Table 2.5, *The Antikythera fragments*) below.

Table 2.5. The Antikythera fragments

Antikythera mechanism fragments

Antikythera main fragment ~150–80 BC[143]

Antikythera dial fragment

By turning the wheels on this device with a handle the user was able to determine

140. Philip Ball, "Complex clock combines calendars", *Nature*, vol 454, published online 30 July 2008, p. 561, `http://www.nature.com.ezp.lib.unimelb.edu.au/news/2008/080730/full/` (viewed 14 October 2011)
141. T. Freeth, Y. Bitsakis, X. Moussas, J.H. Seiradakis, A.Tselikas, E. Magkou, M. Zafeiropoulou, R. Hadland, D. Bate, A. Ramsey, M. Allen, A. Crawley, P. Hockley, T. Malzbender, D. Gelb, W. Ambrisco and M.G. Edmunds, "Decoding the ancient Greek astronomical calculator known as the Antikythera Mechanism", *Nature* vol 444, 2006, pp. 587–591

the relative positions of Sun and Moon. The lower back dial predicted luna eclipses whilst the upper dial was a Metonic calendar (based on the 19 year cycle the Moon takes, seen from a particular place on Earth, to return to the same place in the sky).[144] All 12 months of the calendar have now been identified in relation to the device.

The Antikythera was of Corinthian origin, suggesting a heritage going back to Archimedes (who died in 212 BC). In support of this contention in 1974 Derek J De Solla Price pointed out that Cicero (106–43 BC) wrote that Archimedes built a planetarium which showed the Moon rise following the Sun above the Earth, together with eclipses and the motions of the five known planets, suggesting that the Antikythera had a similar purpose. However, the extent to which the Planetarium could have performed these claimed tasks has been open to controversy.[145]

The purposes of the Antikythera in any case should not be considered solely astronomical. An upper minor dial follows the four-year cycle of the Olympiad - the most important of the associated Panhellenic games. Thus the device, identified as being from about 100 BC, was "not simply an instrument of abstract science, but exhibited astronomical phenomena in relation to Greek social institutions."[146]

A chasm exists in sophistication between this high-precision geared calculating device and the pebble accounting described earlier. Whilst much is unknown about it it is clear that the development and production of the Antikythera machine would have required high mathematical understanding, and the most sophisticated artisanal skills available. It is likely it was produced for a person or institution of the highest standing in Greek society who was able to call upon the most advanced talents of the day. The extraordinary accomplishments embodied in it tells us much about what could be achieved in the sophisticated society of ancient Greece in the later part of the "Golden Era" of Greek mathematical invention. But as well, this illustrates the dangers of assuming a sharp division between pragmatic and abstract mathematical cultures, or useful versus theoretical outcomes. This accomplishment clearly drew on high skills in both areas. Above all it is a reminder of the difference between perceived social need, and innovation - where invention actually finds use in practice. After all, where pebble accounting will do, why use anything more complicated and expensive?

This relationship between perceived need and innovation is strikingly illustrated by the fact that the principle of the steam engine was demonstrated in ancient Greece, in the form of the aeolipile (or "Hero's engine"), more than two thousand years before the industrial revolution. As described by Hero of Alexandria, a hollow ball was caused to spin by steam blown out through two counterposed jets.[147] It may well have been possible with the known technology at the time to extend this method to

144. See for example, `http://www.astrocal.co.uk/metonic-cycle.htm` (viewed 25 Jan 2012)
145. See, for example, "Archimede's Planetarium", *Museo Galilleo*, `http://brunelleschi.imss.fi.it/vitrum/evtr.asp?c=8253` (viewed 25 Jan 2012).
146. Tony Freeth, Alexander Jones, John M. Steele and Yanis Bitsakis, "Calendars with Olympiad display and eclipse prediction on the Antikythera Mechanism", *Nature*, vol 456, 31 July 2008, pp. 614–617.
147. The Pneumatics of Hero of Alexandria, Translated for and Edited by Bennet Woodcraft, Taylor Walton and Maberly, London, 1851. Reproduced at `http://www.history.rochester.edu/steam/hero/section50.html` (viewed 25 Jan 2012)

create a working steam powered machine. But it is likely there would have been little incentive to explore this possibility since there was plenty of slave labour available to the free citizens of the ancient Greek world. Indeed in the same tract in which the steam device is described there is serious attention paid to the perceived value of applying practical applications of pneumatic devices to producing mysterious effects for use in religious ceremonies.[148]

As the above suggests, apparently obvious developments may be ignored where no need for them is perceived. However, where power, authority or ceremony is seen to require a state of the art development, and sufficient resources are made available, then the best minds and most skilled talent may be brought to the task. It is not unreasonable to speculate that as with the success of the Manhattan and Moon Landing projects of the Twentieth Century (CE), some two thousand years before there were some of high authority also able to martial the best talents available. They too could create an object of technological achievement that would be a source of wonder for all privileged to view it. But whether it was through that means or the work of a School led by a Master who brought together the high artisanal, mathematical and empirical knowledge required, what is known is that for the time an extraordinarily refined analogue mechanical calculating device was made two millennia ago. The surviving evidence of that lies in the eroded but still comprehensible remains of the Antikythera mechanism.

Building on multiple pasts - the "Arabic hegemony"

As already noted, with the fall of the Western Roman empire around 500 AD, mathematical development from the ancient world had ceased.[149] From the death of the prophet Mohammed at Medina in 632 AD there was a rapid expansion of the Islamic state which by 641 AD had included in its conquest much of Mesopotania and the former center of mathematical learning at Alexandria. The conquerers had little interest in mathematics. To the extent that they enjoyed collective unity it was primarily through economic exchange and shared tenets of the Islamic religion.

The following century was characterised by internal warfare and political turmoil. That began to subside around 750 AD. Following that there commenced a re-discovery of lost mathematical and scientific knowledge. Informally a process of diffusion of Indian, Greek and Persian knowledge had begun during the preceding century.[150] A more formal and systematic process now began focussed on Baghdad under the three powerful patrons of learning - the caliphs al-Mansur (754–75 AD), Haroun al-Raschid (763–809 AD) and al-Mamun (812–33 AD). There al-Mamun, also regarded as the

148. see index, ibid, `http://www.history.rochester.edu/steam/hero/index.html` (viewed 25 Jan 2012)
149. Except where otherwise noted, all of the detail in this section (including the helpfully descriptive phrase "Arabic hegemony" is drawn from the excellent treatment in Boyer and Merzbach *A History of Mathematics*, Chapter 13, pp. 225–45.
150. François Charette, "The Locales of Islamic Astronomical Instrumentation", *History of Science*, xliv, 2006, p. 124

"glorious initiator" of the "translation movement", established a "House of Wisdom" not unlike the prior Library of Alexandria.[151] Once more the detail of what was rediscovered and further developments from that is described in detail elsewhere.[152] It is sufficient to note here that Baghdad became a cosmopolitan centre of learning. But following Baghdad, over time the Arab world - comprising as it did a wide range of communities with diverse histories, backgrounds and cultures (including the Egyptian and Greek peoples) including also not only Muslims, but also Jews and Christians - provided a fertile background against which the desire to recapture and build on lost knowledge could be realised.

This civilisation took what it deemed necessary from the prior received knowledge. In relation to numerals the Indian system of discrete Hindu numerals, ordered in their places according to powers of ten, and with the zero included, slowly gained precedence. By the 16th Century it was giving rise to the Modern European system of "Indian-Arabic" numerals described earlier.[153] Over the period from the eighth to the 15th centuries arabic mathematics developed to include:

- arithmetic primarily derived from India (including the principle of position);
- geometry rediscovered from ancient Greece but extended with a few additional general observations;
- trigonometry primarily from ancient Greece, but to which Hindu form, and additional functions and formulas were added, and with the most innovation; and
- algebra derived from Greek, Hindu and Mesopotanian sources but assuming a new and systematic form[154] albeit still not expressed in the economical symbolic form that would later be created by Descartes, in the Modern era. (The term "algebra" derives from a book titled *Kitab al-Jabr wal-Muqabala*, published in the early ninth century by the famous mathematician and member of al-Mamun's scientific academy, al-Khwarizmi, a page of which is shown in figure 2.19 below. It is the oldest known Arabic work on algebra.[155])

The last of a line of great Muslim mathematicians of this period (who died in 1436), Al-Kashi, amongst his various achievements, calculated pi to 14 decimal places (in both sexagesimal and decimal forms), a notable feat not matched again until the late sixteenth century after the re-emergence of mathematical development in Europe.[156]

As to the technologies of calculation, the Islamic world had access to the counting boards, dustboards (used particularly in India), and abacus of the other civilisations that they had incorporated. One Damascus mathematician, al-Uqlidisi is known to have adapted the Indian system for pen and paper. Amongst the many advantages of this were the permanent recording and transmission of calculations using the

151. ibid, p. 124
152. Boyer and Merzbach *A History of Mathematics*, pp. 225–45.
153. ibid p. 237
154. ibid p. 240
155. John L. Esposito (ed), The Oxford History of Islam, Oxford University Press, UK, 1999, p 186.
156. ibid pp. 244–5

Figure 2.19. Page from the Compendious Book on Calculation by Completion and Balancing (Kitab al-Jabr wal-Muqabala) by al-Khwarizmi: ~780–850 AD

efficient Indian notation.[157] By this means the sharing of problems, equations, and methods for solving them was greatly facilitated and the basis for rapid development in a systematic form of mathematics, and particularly algebra (the word itself being derived from the Arabic), was established.

As already mentioned the measurement and prediction of the movement of celestial bodies has been central to many aspects of ancient life. These range across their astrological and religious implications, the timing of ceremonies, and the prediction of seasonal changes for agricultural purposes. This was no less true of the Islamic world where celestial observation was required to time the call to prayer, to determine the direction of Mecca, and to inform the work of astrologers. From the program supported by al-Mamun the Ptolmaic view of the universe became adopted everywhere in the Islamic world. The instruments named in the *Almagest* such as the gnomon, sun dial and quadrant, and armillary sphere were constructed and utilised. It was the C10 bilibliographer Ibn al Nadim who reported that al-Fazari was the first Muslim to have made an astrolabe.[158] The astrolabe diffused through the Islamic world. Many of the most elegant were crafted under the direction of leading scholars who appeared, often with globes, in illustrations. These instruments could be found

157. `http://www.islamicspain.tv/Arts-and-Science/The-Culture-of-Al-Andalus/Mathematics.htm`
158. Ibn al-Nadim (ed. by R. Tajaddud), *Ktab al-Fihrist*, Tehran, 1971, p. 332 (cited in Charette, *Locales*, p.124).

in many of the royal libraries as well as in the observatories which were constructed to systematise observations of the skies.

Of course the documentation of the history of mathematics and science is biased inevitably towards those areas that would have been recorded. Access to this would have been highly restricted, and was most likely to be found in Courts and Mosques. Certainly royal patronage was crucial in the early stages of reconstruction of ancient knowledge and in supporting the development of instruments and facilities of astronomical observation. This led to a pattern reminiscent of the "two cultures" already referred to. As noted earlier a division between "two cultures" of theoretical and pragmatic mathematics could be seen in the Athenian state of the Classical period. For the free citizenry forming the elite of this period (well serviced as they were by slaves and other non-citizens) the artisan or other person who performed work was thereby subordinated to the user of the products of that labour. Thus for Aristotle and Plato the free man is a user, never a producer. Ideally he uses things correctly but never transforms them by work.[159] This explains in part the highly abstract presentation of mathematics by the Classical Greek mathematicians.

Similarly, the mathematics or science that was developed, or the instruments prepared for Court astronomy in the Arabic world, frequently had little of what might normally be considered practical application. Whether amongst the free elite in the slave society of ancient Greece, or the royal courts of the Arabic world, the actual doing of constructive work was something which the elite distanced themselves from and left to others. So the motivations for technical progress tended to either be the hope of attracting royal patronage, or ultimately not merely to satisfy a desire to advance knowledge but by being seen to do so to confirm the glory and wisdom of royalty who supported it. Having said that, much of great abstract value was accomplished through these means. Major developments included the construction of large observatories (one with a mural quadrant twenty metres in diameter) and the provision of employment to the most expert instrument makers.[160]

Further, as Charette points out, in a particularly valuable article, this discussion of usefulness tends to ignore another important use. That was the usefulness of science and scientific instruments for religious practice (as well as more mundane activities). Several leading astronomers (including the legendary al-Khwarizmi and Bayt al-Hikma) applied astronomical tables to the determination of prayer times and for predicting the religiously important first sighting of the lunar crescent. They laid out methods for finding the direction of Mecca, the construction of water clocks, sundials and hororaries, and the designs for sine quadrants and instruments for predicting lunar eclipses.[161]

Charette notes that from the middle of the C10, the progressive disintegration of the Abbasid empire led to a proliferation of local dynasties and principalities.

159. Herbert A. Applebaum, *The Concept of Work: Ancient, Medieval and Modern*, State University of New York Press, Albany, 1992, p. 31
160. Charette, "The Locales", p. 125.
161. ibid, p. 126.

This provided greater opportunity, in a diversity of places, in which patronage for astronomical and mathematical development might be found. Other centres such as Cairo and Damascus were also centers of such work over C13-C16. Whilst over this time the emphasis of patronage of such work in Courts diminished, the practical usefulness of the work in religious observance was picked up. As a consequence there was a diffusion (and to some extent rediscovery) of mathematical and astronomical knowledge and instrument making from Courts to Mosques. The boundary between Mosque and Public was in any case indistinct. These prized instruments, constructed in private shops, could been purchased by citizen and Mosque alike.[162] By C15, the knowledge and possession of astronomical instruments such as the astrolabe, and the practical utility of it for determining time and direction, had become diffused more generally through Islamic society. However, the Islamic world was now shrinking under military pressure from Western Europe. It was being forced out of Spain, and the Renaissance was under way in the rising powers of Europe. This Renaissance was now dominated by the religion of Christianity, but nevertheless drew heavily on the Islamic world's trove of rediscovered ancient knowledge together with the Islamic world's own discoveries in mathematical and scientific knowledge, and invention of calculational and observational instruments.[163]

Dimensions of development

Enough has been said in what is inevitably a panoramic rather than detailed survey to suggest some (interconnected) factors that have shaped the direction and extent of development of different calculating technologies. These include the:

- *value a society places on innovation and the existing stock of knowledge.* The above history reflects an apparent rise and decline of social commitment to innovation together with the loss and rediscovery of past insights and developments. This can occur for different groups in society or for societies as a whole. The "Dark Ages" in Europe following the collapse of the Western Roman Empire (C5-C15) is an example of a time when mathematical innovation apparently lost much of its lustre and prior technical knowledge was, for a time, lost.
- *complexity of social organisation* and in particular the scale of hierarchical organisation and power and consequent organisation needs. With the extensive engineering works in ancient Mesopotania, and its more complex agricultural society, there was a need for more systematic means of calculation and development of approaches to meeting that need.
- *relationship between perceptions of what needs to be done, and available or realisable technologies of calculation.* This relationship goes both ways. The invention of new calculational approaches (for example of more efficient

162. ibid. pp. 128–132
163. Alison Abbott, "Islamic Science: Rebuilding the Past", *Nature*, Vol 432, 16 Dec 2004, pp. 795–5.

numeral systems, or of a system of geometry) also opens the possibility of new social needs emerging.

- *dynamic interaction between mathematics and calculational technology.* Developments in each of these depends on the developments in the other. As illustrated by the case of Roman numerals, even the form of numerals developed to write down quantities in part is shaped by the medium in which they are to be inscribed (for example, stone or parchment), the needs of the society to use them, and the technologies (such as pebbles or abacus) available to assist in manipulating them.
- *relationship with users.* From the Classical Greek period on we see the division emerging between the roles of calculation in the Courts, or the elites, which can lead to a distancing of developments from practical application needs, and the needs and capacities in the broader community. Instruments which are part of the play for patronage, or part of the process of celebrating power and standing may take a very different form, and play a very different role to those used as part of every day social interchange.
- *relationship with producers* of calculating technologies and their social position. Where artisans are required to develop such instruments they may play a very different social role to the users. The classical Greek divergence between theoretical mathematics and pragmatic artisanal mathematics has been referred to. Artisans not only have a social role, but also an accumulated knowledge that may be in part be encapsulated in embodied skills, and may routinely be transferred in non-codified forms. In various periods of history the guild-apprenticeship model has been the method of development of transference of this knowledge, the bulk or all of this being handed on through a mixture of oral instruction, and learned skills. Developments in this process may be directed much more sharply to solving perceived practical problems and may lead to quite different emphases and outcomes from those in more abstract and theoretical environments.
- *extent of communication* with other societies. Trade and other forms of communication with other societies enlarges the access to new knowledge, skills, and technological developments. The diffusion of Islamic knowledge (for example, about astronomy) to Europe and India, as well as Indian knowledge (and in particular the Islamic numeric script) to the Islamic world, provide examples of this.
- *role of authority and power.* The driving force of systems of authority, from the Egyptian pharonic society to the Islamic Caliphates, to the replacement of the Inca system of double accounting via the Khipu by the Spanish conquerers with European approaches,[164] the role of authority and, as already mentioned, patronage, in shaping calculational approaches and technologies, is evident.
- *breadth of need and literacy.* Where the need for calculation is confined to a small subsection of society (for example, to priests of the dominant religion) then the need may be able to be met by training in traditional methods. But

164. See Robsan and Stedall, *The Oxford Handbook*, Chapter 1.2, pp. 27–55.

where the need spreads (for example, with trade and intensifying markets) then calculational technologies may offset the need for intense training in numerical and mathematical literacy. On the other hand, there is no need for technologies of calculation where there is no capacity to manipulate numbers. Thus the modes of education, whom they served and the breadth of their reach, and finally the extent of their numerical and mathematical content, will determine the usefulness, development, manufacture, and use of particular calculation instruments and approaches.

- *role of specialisation.* Specialised approaches such as those described earlier to measure and calculate astronomical or astrological phenomena may drive a particular direction of development which has little more general application, even though it may have very profound implications for some key processes. For example, the developments leading to the astrolabe was certainly of great importance for religious life and ultimately also for the organisation of social time and support of navigation and trade had great implications for society. But whilst associated developments in mathematics may have had many spin-offs, they were of a different character to those which directly seek to advance the overall power of mathematical calculation in a general way.
- *cost and accessibility* of calculational technology. Finally the utility of a calculating technology will be determined in part by its cost and accessibility. This will be determined by the extent that the problems it can solve are seen to be widely experienced, the training and literacy required to use it, and its cost of production. The use of instruments like the astrolabe or Roman abacus, let alone the Antikythera, were clearly circumscribed by what was required to construct them, in contrast, for example, to the use of pebbles for reckoning.

Consideration of the development of technologies of calculation in the Pre-Modern period suggests that these factors are helpful in understanding why particular technologies were developed and used at different times. As will be seen in Part 2 (next), they are also very useful in understanding what has shaped the directions of development in the dynamic Modern period which followed.

Chapter 3

The Modern Epoch and the Emergence of the Modern Calculator

3.1 Part 2. The Modern Era

The "Modern Era" (or the "Modern Epoch") is taken as stretching from the mid-sixteenth to mid-twentieth centuries.[1] This was a period of accelerating change in Europe and, amongst much else, a new burst of innovation in mathematics and mechanical calculating technology.

Even by the late sixteenth century in practice methods of calculation had not changed much since the Roman Empire. Indicative of this "calculi" (Latin: "calculus" for "pebble" (or "limestone") - the origin of the word "calculation") were still being struck (as they had been for centuries) to facilitate counting and arithmetic especially in many fields of commerce. They were referred to differently in different places. One such - a French "jeton" from 1480–1520 is shown in figure 3.1 below. It is made to look like a coin, but its inscription is meaningless.

Figure 3.1. Sides 1 & 2, French jeton 1480–1520
(collection Calculant)

Sometimes these calculi (however named, and in whatever form they took) would be "cast" on a "counting board". Sometimes they would be simply piled up to aid the process of accounting or calculation. Their use petered out in England over the C18 and similarly in France ending with the revolution (1789).[3] There was in particular a contest between the incoming technology of pen and paper using arabic numerals, and calculi and counting board based on Roman numerals. This contest is shown in the following woodcut (figure 3.2) from 1503,[4] where, presided over by Dame

1. *Joseph Camilleri and Jim Falk, Worlds in Transition: Evolving Governance Across a Stressed Planet, Edward Elgar, UK, 2009*[2]
3. Francis Pierrepont Barnard, *The Casting-Counter and the Counting-Board: Chapter in the History of Numismatics and Early Arithmetic*, Clarendon Press, Oxford, 1916, p. 87.
4. From Gregorius Reisch, *Margarita philosophica nova Anastatic*, 1503, reprint with an introduction (in Italian) by Lucia Andreini, Salzburg: Institut für Anglistik und Amerikanistik, Universität Salzburg, 2002 (3 vol.).

Arithmetic, on the left Pythagoras is using the new technology, whilst on the right the ancient Roman mathematician Boethius is calculating with counting board and calculi.

Figure 3.2. Contested methods - Woodcut from 1503

For such counting boards the horizontal lines represent rows of multiples of 1, 10, 100, 1000 (or in Roman numerals I, X, C, M), whilst the mid point between those lines represent the half-way mark of 5 (V), 15 (XV), 50 (L), 500 (D) and 1500 (MD). For addition the number on the left could be progressively added to the number on the right by rows. In the woodcut, Boethius is adding the number MCCXXXXI (1,241) on his left (our right), to the number LXXXII (82) on his right. (Multiplication could be done by repeated additions.[5])

As the use of calculi reached its zenith they became important in other ways. The example minted in the Netherlands in 1577 shown below (table 3.1, *Calculi*) demonstrates that by then, counters were in such general use that they were being used to convey messages to the public - becoming the equivalent of official illustrated pamphlets.[6] The face on the left ("CONCORDIA. 1577. CVM PIETATE") translates

5. see for example, Dan K. Bell, "Calculating with Calculi: the Counting Board and Its Use in Reckoning in Medieval Europe", *Proceedings of the AMATYC 31st Annual Conference*, San Diego, California, 2005, pp. 20–35. `http://www.amatyc.org/`, viewed 10 July 2013.
6. Ibid pp. 40–49.

(loosely) to "Agreement. 1577. With Piety" and seems to carry the visual message that we are joined together (with hearts and hands) under our ruler (the crown) and God (represented by a host). The opposite face ("CALCVLI.ORDINVM.BELGII. Port Salu") shown on the right) translates to "Calculi. The United Provinces of the Netherlands, Port Salu" and shows a ship (perhaps the ship of state)[7] coming into Port. (The term "port salu" at the time often simply meant *"safe harbour"*[8]). It was not a calm time. It was in the midst of the European wars of religion (~1524–1648), and in particular the Eighty Years' War in the Low Countries (1568–1648). It was only 28 years after Holy Roman Emperor Charles V had proclaimed the Pragmatic Sanction of 1549 which established "the Seventeen Provinces" as a separate entity. (This comprised what with minor exception is now the Netherlands, Belgium and Luxembourg.) Given that there had been a revolt in 1569 (leading to the Southern provinces becoming subject to Spain in 1579), only two years after this calculi was minted, we may presume that the messages of widespread agreement, safe harbour and rule from God might have been intended as a calming political message.

Table 3.1. Calculi

	
CONCORDIA.1577.CVM.PIETATE.	CALCVLI.ORDINVM.BELGII. PORT SALV
1577: Sides 1 & 2 of calculi	(collection Calculant)

The system of "pebble" or "ocular arithmetic" using counters stretched back seemingly into the mists of pre-history. But in the middle of the sixteenth century a process of dramatic change was beginning. It heralded the re-growth of political power in Europe and Britain. Over the next two centuries this would be accompanied by a dynamic development in knowledge, trade and innovation. By the beginning of the eighteenth century the industrial revolution in the "West" would be well underway. Drawing from earlier innovation in the "East" this now outpaced it in terms of economic and political power and technological innovation. It was a time

7. Barnard, *The Casting-Counter*, jeton 69, p. 203.
8. `metastudies.net/pmwiki/pmwiki.php?n=Site.Calculi1577`

of increasing fascination with new knowledge and mechanisation, continuing rise in scale and economic importance of cities, growth of what became industrial capitalism. Increasing dominance of institutions of the market (including the modern corporation) shaped also new products produced by increasingly sophisticated technological means. In support of this, availability of paper and literacy spread. Roman numerals were increasingly replaced by their Arab-Indian counterparts in every-day use, and the reliance on "ocular arithmetic" with counters was on the decline. Developments in mathematics and calculation, although important, were just one component of this remarkable transition.

3.1.1 Change & the Modern Era

Central to this period of change was the extent to which the feudal system of organisation, that had dominated life in Europe and Britain for more than a millennium, was now being undermined. From the thirteenth to sixteenth century improvement in agricultural practices (for example, the introduction of the three-field system and improvements in ploughing) had allowed much more food to be produced from the same land. With less labour devoted to agriculture there had been more available for diversification into production and trade in an increasing variety of commodities. This marked the first phase of the transition from feudalism to an increasingly dominant capitalist economic and political system.

The feudal system of land had been controlled through a system of manors by feudal secular and religious lords (and more elevated nobles). Church and state supported a view of the feudal order as natural and immutable - one in which lords and serfs performed enduring roles within a system of mutual obligation. As agriculture became more efficient increasing numbers of "free men" with greater social mobility began to challenge the entrenched ways and power of the feudal system. Freed from the obligation of labouring on the land, as early as the eleventh and twelfth centuries, these freemen began to find new work as merchants or in other productive occupations first in towns, and then large industrial towns. Unshackled also from the manorial system their allegiance was more directly to kings (queens and princes) rather than lords. As merchants became more numerous they increasingly gained the concessions from the Royal courts necessary to carry out ever more sophisticated forms of commerce.[9]

The effect of the burgeoning new economic transactions, supported by new rights and laws, was to increase both the power of merchants and the royal courts (initially at least) in comparison to that exercised by the feudal lords. Ownership of the means of production (land, buildings and technology) was passing to the hands of a new economic class. The richer owners of land bought up the land of others enabling them to apply improvements such as the use of fertilisers and specialisation in crops and animals. At the same time the factory system was emerging, first as a form of control,

9. for an accessible rendition of this see, for example, E. K. Hunt and Howard J. Sherman, *Economics: Introduction to Traditional and Radical Views*, Second Edition, Harper & Row, New York, 1972.

then as a place for deploying new technologies of production. At first, merchants moved from simply selling the product of the peasants' labour, to organising its production and selling it. For example, in the case of woollen cloth, rich merchants began to "put out" orders to peasants for the wool. Peasant weavers and sewers were gathered into employment in factories where cloth would be made, or fashioned into clothing. Richer merchants began to commission the application of new technologies in their factories, to cheapen and speed production.

By the late eighteenth century and into the nineteenth new technologies were shaping commerce. These included the telegraph, steam power, the railway, and the development of the factory system now powered with such technologies. The feudal system became increasingly submerged by the ever more dynamic, productive and powerful force of the industrial revolution.[10] Trade was increasing not only in volume but also reach. New technologies of navigation and shipping were resonating with new means of production, forms of transportation, and ways of transmitting information. Whilst in 1750 it had taken as long to travel or send information from one place to another as in the ancient Greek or Roman empires, by the end of the following century travel by railway across great distances was becoming vastly faster, and information could be sent by telegraph nearly instantaneously. Factories, trade and cities all expanded as the needs of the new system were met and fed. The celebration of technology, and use of it for all aspects of this transition to industrial production, was becoming a central tenet of Modern life.

3.1.2 Strands of change

This turbulent time was both a fertile ground for the development of new knowledge (including mathematics), and the application of that knowledge to the practical work of production (including technologies of calculation). However, diverse developments were occurring, across different parts of the societies. Perhaps confusingly all of these strands were to some extent intertwined. Some of the more important of them are discussed below.

Aristocrats, artisans and the rekindling of critical inquiry

Key to the spirit of change that was taking place was a renewed fascination with learning and innovation. The certainties that had supported the established feudal order were, from the fourteenth century and over the next three centuries, increasingly challenged. The approaches which eventually became known as "science" (in its Modern sense) were beginning to be promoted and gain support. What began to be proposed and then taken up was a systematic and incremental process of discovery based on the practical investigation of empirically testable hypotheses built on the basis of prior work, the whole being subject to critical peer response.

10. Hunt and Sherman, *Economics*, pp. 52–3.

It has been estimated that in 1668 in England "the temporal and spiritual lords, baronets, knights, esquires, gentlemen, and persons in offices, sciences, and liberal arts" together represented about 4% of the population,[11] (although enjoying about 23% of the income).[12] Clearly those associated with the sciences in general, and mathematical work in particular, constituted only a tiny fraction of this. It may have been an initially small group of people involved, but as inevitably it was drawn from those who could afford the 'time out' to devote themselves in this work, drawn as they often were from aristocratic families, they had the capacity to convey their excitement at new insights amongst those of standing, and not least to reach the ears of royalty.

The role of the emerging practice of science became particularly confrontational for religious authorities (and particularly the powerful Catholic Church). Astronomical observation and theory had long played an important role in human life. Apart from its traditional role in astrological prognostication, astronomy could be turned to the prediction of seasonal changes such as tides, and the fixing of time and position. However, religious orthodoxy was the Ptolemaic concept that the Earth lay at the centre of the universe with the planets, sun and stars revolving around it in concentric spheres. The seminal work of Nicolaus Copernicus *De revolutionibus orbium coelestium (On the Revolutions of the Heavenly Spheres)* was published in 1543 (just before his death) providing a systematic justification of a view of the solar system in which the planets, including the Earth, revolved around the sun. This sparked a major theological and scientific controversy. Whilst Copernicus had the planets moving in circles, it remained for the German mathematician, astrologer and astronomer, Johannes Kepler, in 1609, to publish mathematical arguments showing (amongst other important insights) that a much simpler explanation was that the planets move in ellipses.[13] Steadily the convergence of observation and mathematical insight was bringing astronomy from an adjunct to philosophical speculation and theological dogma, to a science of the motion of the heavenly bodies. It was not however, until 1758, that the Catholic Pope of the time removed Copernicus's book from the index of forbidden reading.

The desire to predict the motion of heavenly bodies in itself provided demand for not only more powerful mathematical insights, but also aids to calculation. Astronomical investigation was based on observations of planets and stars where the calculated paths were redicted from a theory which involved not circular cycles, but also epicycles and ellipses. Making such predictions required a vast number of calculations involving repetitive additions and multiplications. New ways would soon be developed which could help with this.

In addition, despite theological concern associated with the rekindling of scientific interest, "the idea of progress" was finding particular favour with the increasingly powerful merchant class. An underlying promise here was that, rather than awaiting one's rewards for the afterlife, technical and industrial development could increas-

11. estimated in terms of the percentage of families.
12. Carlo M. Cipolla, *Before the Industrial Revolution: European Society and Economy, 1000–1700*, W.W. Norton and Company, USA, 1976, p.13.
13. Johannes Kepler, *Astronomia Nova*, 1609

ingly be relied on to satisfy needs in the here and now.[14] "The idea of progress" stressed a claimed progressive character to science. Over time, seized on by the emerging entrepreneurial class it came to be part of an argument for the new order, built around the market, to be given greater freedom to act, and corresponding political standing. As a consequence, as Bury puts it, during this period increasingly "Self-confidence was restored to human reason, and life on this planet was recognised as possessing a value independent of any hopes or fears connected with a life beyond the grave."[15]

The idea of progress was an ideology which could give legitimacy to the claims of a particular emerging class. But it was based on developments in thinking that were led by a comparatively small set of intellectuals. The questions they worked on often would have seemed quite divorced from everyday life. These mathematical and other scientific pioneers were frequently drawn from the aristocracy or church, or at least were gentlemen of considerable independent means. The motivations for doing this work might be scattered across a spectrum. It could include: a delight in learning and discovery, a desire to build prestige amongst peers, a hope for economic return from practical applications, or a desire to find favour with a rich or royal patron. Over time an increasing number of kings, queens, and other nobles began to enjoy being seen as a supporter of progress, or became interested in the work of intellectual pioneers.

A gulf still stood between the discoveries by mathematicians and other intellectuals, and the many others to whom this work could be of practical assistance. On the one side of that gulf, reflecting upon these early intellectual innovators, was the long-standing idea that a man of elevated (or aristocratic) heritage - a "gentleman" or in France "un honnête homme" - would consider it demeaning (as would an ancient Greek or Roman of standing some 1500 years before) to lower himself to associate himself with practical work. On the other side, amongst those whose life was devoted to practical work (for example, artisans) a parallel image was common, of the impractical nature of the gentleman mathematician and the products of mathematical thinking. This gulf would have to be surmounted before these the technical insights of these "gentlemen" could be turned to widespread practical purpose.

The beginnings of the renewal of mathematical and scientific learning thus did not amount to a neat picture. Old ways of doing things lay not just with the aristocracy. As Spencer Jones points out,[16] even into the seventeenth century whilst learned men began to press forward mathematics, navigation remained "a practical art, in which successes depended upon experience, common sense and good seamanship. The navigator had for his use the compass, the log, and some sort of cross staff" with which, together with his estimate of wind speed and currents, he would estimate his position by a "crude method of dead reckoning".

14. for a much more sophisticated rendition of the history of this concept see Bury, *The Idea of Progress.*
15. J.B. Bury, *The Idea of Progress: An Inquiry Into Its Origin and Growth*, 1920, reprinted, The Echo Library, UK, 2010, p. 23
16. H. Spencer Jones, "Foreword by The Astronomer Royal", in E. G. R. Taylor,*The Mathematical Practioners of Tudor & Stuart England 1485–1714*, Cambridge University Press, Cambridge, UK, 1954, p. ix.

Mathematics was not taught in schools, and where it was pursued, as with the rest of science, it tended to be the pursuit of those with sufficient (often inherited, or previously accumulated) resources to do so. Thus, as one writer, in 1701, put it:

> The great objection that is made against the Necessity of Mathematics in the... great affairs of Navigation, the Military Art, etc., is that we see those affairs carry'd on and managed by those who are not great Mathematicians: as Seamen, Engineers, Surveyors, Gaugers, Clock-makers, Glass-grinders., and that the Mathematicians are commonly Speculative, Retir'd, Studious Men that are not for an active Life and Business, but content themselves to sit in their studies and pore over a Scheme or Calculation.[17]

Nevertheless, over time, the usefulness of technical developments, including the outcomes of mathematical calculation in astronomy, engineering, and commerce would break through the barriers of traditional practice. And despite the reticence still evident in 1701, the pressure to compete effectively in trade and warfare would increasingly lead to deliberate efforts to utilise the outcomes of the work of the "Speculative, Retir'd, Studious Men".

Trade, navigation and shipping and the developing economy.

Even prior to the Modern era, the increasing importance of mercantile and thus also naval shipping was reshaping the understanding of needs especially at the level of government. Early developments around navigation and the construction of ships enabled more reliable open sea transport and naval expeditions. By the mid fifteenth century Portuguese ships had rounded the Cape Bojadar on the West Coast of Africa (in 1434), and the quadrant and a few decades later the astrolabe (in about 1480) had come into use.[18]

The second half of the fifteenth century through the sixteenth century was a time of such dramatic European exploration by sea that it is often referred to as "the age of exploration". Notable amongst the European achievements were the charting of sea routes to India, Africa and the Americas. (Christopher Columbus reached America in 1492, whilst Sir Francis Drake claimed San Francisco Bay for Queen Elizabeth in 1579.) As a consequence, there was a large flow of gold and silver, amongst many other commodities, from the Americas to Europe. This was the source of a powerful inrush of wealth and thus investment and purchasing power for those who gained possession of it.[19] Increasingly complex financial techniques were needed

17. J. Arbuthnot, "An Essay on the Usefulness of Mathematical Learning in a Letter from a Gentleman", 25 Nov 1700, quoted in Taylor, *The Mathematical Practioners of Tudor & Stuart England*, p. 3.
18. Carlo M. Cipolla, *Before the Industrial Revolution: European Society and Economy, 1000–1700*, W.W. Norton and Company, USA, 1976, p.167.
19. Hunt and Sherman, *Economics*, pp. 23–4

to take advantage of long-distance trade.[20] This was but an early contribution to the increasingly complex financial flows, instruments and organisations which would be developed in support of, and in order to gain advantage, in the increasingly complex market capitalist economy. This economy would develop over the next several centuries creating ever greater demands for an ever more distributed capacity for efficient calculation.

Innovation requires multiple inventions. Once they are applied in practice this creates new opportunities for innovation creating a dynamic system of change. Increased navigation meant increased trade, requiring increased naval protection of trading routes, requiring improved navigation. The improvement of navigation depended as much on the capacity to print, which Gutenberg had pioneered in 1449, as on new forms of calculation. For example, in sixteenth century England, an early innovation was to replace the oral instruction and reliance on memory which had characterised British navigation at sea, with books of charts, tables and sailing practices, an approach that the Dutch had already pioneered with the *Spiegel der Zeevaert* published in two parts over 1584–5. In England, when a copy was displayed in the Privy Council a decision was made to translate the document and modify it for use in England, with it duly appearing as *The Mariners Mirrour* in 1588.[21] With an acceptance that seafaring could be assisted by printed aides it was only a matter of time for the desire to improve them to create a further demand for more accurate calculation of more useful navigational tables.

Competition at arms

The need for more accurate maps added to the demand for simpler ways of carrying out calculations. The defeat of the Armada at Gravelines during the (undeclared) Anglo-Spanish War of 1585–1604 provided a telling lesson in the sixteenth century of the importance of not only the economic power of merchant shipping, but also of the military importance of naval power. In particular it reinforced the need for manoeuvrable naval ships effectively utilising the best available gunnery.[22]

The use of cannon, muskets and pistols in warfare both on land and sea, had a history stretching back several centuries. But it had become a recognised feature of warfare by the mid-sixteenth century. So much was this so that King Henry VIII found himself troubled by shortage of gunpowder in his invasion of France in 1544 AD and had to import it.[23]

20. Larry Neal, *International Capital Markets in the Age of Reason*, Cambridge University Press, UK, 1990, p. 4
21. Taylor, *The Mathematical Practioners of Tudor & Stuart England*, p. 41.
22. see for example, Aubrey N. Newman, David T. Johnson, P.M. Jones, “The Eighteenth Century”, *Annual Bulletin of Historical Literature* Vol. 69, Issue 1, 1985, 93–109 cited in `http://en.wikipedia.org/wiki/Spanish_Armada#cite_note-25`, viewed 15 April 2012
23. Wayne Cocroft, *Dangerous Energy: The archaeology of gunpowder and military explosives manufacture*, English Heritage, Swindon, 2000, Chapter 1.

The early seventeenth century was a turbulent time comprising widespread conflict and upheaval across Europe. Indeed it was so turbulent as to comprise what some historians have referred to as "the General Crisis",[24] a time of confrontation, and in some places overthrow, of the legitimacy of the existing order, and more broadly, the relationships between state and society.[25] Whether it was more profound than the Hundred Years War between France and England (1348–1453) or the Black Death which almost halved England's population (1348–9)[26] is hardly the point. It was a tumultuous time, and the tumult was widespread.

In England, just as an example, religious and political turmoil included: the schism between Rome and England under King Henry VIII between 1533–40, repression of "Papists" under Queen Elizabeth I (who established the English Protestant Church in 1559 and was declared a heretic by the Pope in 1570), further suppression under King James I, the Gunpowder Plot of 1605 in reaction to the treatment of Catholics, struggle in England between those in the House of Commons and King Charles I which ended in his execution in 1649 following the two English Civil Wars (1642–5 and 1648–9), rule by Oliver Cromwell as Lord Protector from 1653–8, and the subsequent restoration of the monarchy under King Charles II in 1660. But also across Europe, parts of the New World, and even beyond, it was a century of major wars and revolutionary upsurges, fertile ground for furious political machination and contest, and as it would turn out, innovation.

The increasingly widespread use of gun and cannon, in particular, provided a growing practical need to be able to calculate the trajectory of cannon balls and other projectiles. There was thus a demand for improved and more widely accessible methods of estimating all the relevant parameters (for example, matching quantity of powder required to wind, inclination and target, and projectile weight and type). By the early seventeenth century the search to solve these sorts of problems began to result in the development, explication, elaboration, popularisation, and with time, greater use ,of various helpful approaches to making the necessary calculations.

Managing in a more complex state and world

As already mentioned the Modern era was characterised by increasing flows of trade and finance between and within nations. Corresponding to this was the growth of cities, the increasing power of the state and merchants, and the growth across Europe of expanding bureaucracies as states attempted to regulate, control, and facilitate the powerful trends underway. Military conflict added to the pressure to wield collective force across kingdoms. Rulers in turn needed to plan, command, and control the collected forces. Consequently, as the Modern era developed, there emerged a virtual army of clerks, customs officials, excise officers, inspectors, quantity surveyors,

24. notably used by Eric Hobsbawm in two articles: "The Crisis of the Seventeenth Century", *Past and Present*, issue 5 and issue 6, 1954.
25. Hugh Trevor-Roper, "The General Crisis of the Seventeenth Century", *Past and Present*, vol. 16, 1959, p. 51.
26. Hunt and Sherman, *Economics*, p. 21

architects, builders, and then technicians. These were assembled to form the apparatus of states as they sought to shape, manage, and control an ever more complex world. At the same time in the ever more complex organisations of commerce, a similar virtual army of employees was constructed to assist in the achievement of profit.[27]

Whether in the state, or the commercial sector, the need for calculation and the spread of the capacity to calculate became greater. Eventually that need would in part be met by the development of a host of calculational aids. But the pattern of change would not simply be one of invention following developing need. Rather those early insights and inventions aimed at aiding calculation, and even their deployment in practice, initially acted more as a prelude to deployment. It was only over a considerable period of time that the society began to understand that these tools of calculation could play a potentially vital role in the emerging work of state and corporation.

A growing fascination with mechanisation, and its increasing introduction into production.

The first developments in the technology of calculation in Early Modern Europe were perhaps as much cultural as economic in their motivation. In particular they reflected a growing fascination with the use of machinery. From the thirteenth century, windmills and the more reliable waterwheels (both of which had been known of since ancient Rome) began to be turned to an ever wider variety of productive purposes - from the traditional role of crushing grain, to cutting stone, blowing bellows in metal work, sharpening knives and weapons, sawing planks, printing ribbons, dressing leather, rolling copper plate, and much else.[28] The productive value of machinery was becoming more widely recognised, as perhaps also was the enjoyment of the way it could extend human capacity and power. Indeed there is no reason to think that it would have been any less rewarding an experience to show off one's new adaption of a water mill than it is to show off a new car or electronic device now. A quote from Walker (if set in pre-feminist language) sums up the way in which innovation is a product of a whole cultural as well as economic and technological system:

> Because we see the machine reshaping society and changing man's habits and way of life, we are apt to conclude that the machine is, so to speak, an autonomous force that determines the social superstructure. In fact, things happened the other way around… the reason why the machine originated in Europe is to be found in human terms. Before men could evolve and apply the machine as a social phenomenon they had to become mechanics.[29]

If calculation was to be used in practice, it had first to be understood to be useful in practical life. Similarly, if calculators were to be developed, then mechanisation

27. for more on this see Camilleri and Falk, *Worlds in Transition*
28. Cipolla, *Before the Industrial Revolution*, pp. 150–65.
29. P. G. Walker, "The Origins of the Machine Age", *History Today*, Vol. 16, 1966, pp. 591–92, cited in Cipolla, *Before the Industrial Revolution*, p. 171.

had to be seen to be both appealing and potentially applicable. In relation to calculation, one key cultural development prefiguring its mechanisation, was the growing fascination with clocks and clockwork.

Cipolla points out that near the end of the thirteenth century people not only had a social use for knowing the hour, but could grasp the possibility and advantages of mechanising the measurement of time because they had already had experience with the mechanical extraction of work from wind and water. By the middle of the fifteenth century increasingly reliable mechanisms had been developed for clocks. Some became show-pieces in town towers. Over the next several hundred years clocks were miniaturised and clockwork became a mechanism for driving clocks, watches, and later music machines, dancing figures, moving pictures, and much more. So pervasive was the impact of clockwork that the universe and even the human body were reconceived as machines, of which God was the ultimate "clockmaker".[30] It will hardly come as a surprise, therefore, that we find that the first artisans to be employed by mathematical instrument makers would be clockmakers, and that the first calculator (as will be described later) was called a "calculating clock".

3.1.3 Scaling the heights: New insights and proportional instruments

As the above suggests, a significant early pressure for assistance in calculation came from the combination of the need for more accurate navigation and the corresponding demand for better astronomical observation. Instruments such as the astrolabe and cross staff were used to measure the positions of the sun and stars. Charts and various tabulated information were used then to calculate the relationship between position and the observations. To gain greater accuracy errors in projection, parallax errors in observation, and the like began to be taken into account. The need for more accurate maps increased demand for finer capacity to draw to scale. Over the C17-C19 a steady process of innovation led to improved instruments assisting observation and calculation to meet these demands.

The measurement of distance had been aided by dividers (or "compasses") at least since the Roman era and now instruments were developed building on that idea. A print of a drawing by Thomas Jefferys (~1710–1771) which shows a range of such instruments from the C18 is shown below (figure 3.3).

In this drawing, at the top can be seen the emerging shape of various compasses, constructed from brass and steel. Below, shown in figure 3.4 from this collection, is a typical pair of such dividers, also from the eighteenth century.

At the bottom left of this Jeffries print can be seen another device - a pair of dividers with a moveable central pivot. This instrument created a means of drawing distances to a particular scale. With one end marking an actual distance, the other could be reduced by a desired ratio by moving the position of the central pivot. An early reference to such a "proportional compass" was made by architect Daniel Speckle, as

30. ibid, pp. 170–2

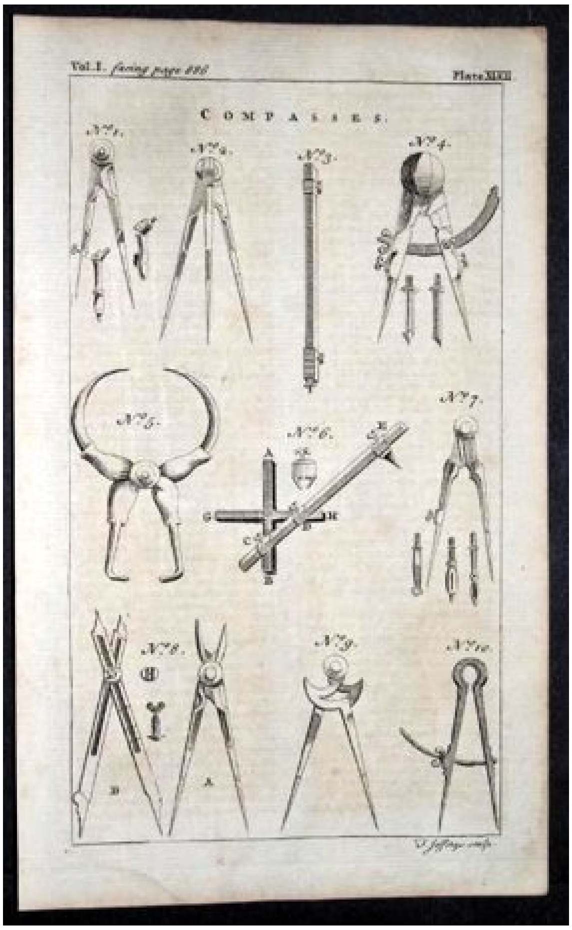


Figure 3.3. 1710–71: Compasses by T. Jefferys sculp (Collection Calculant)

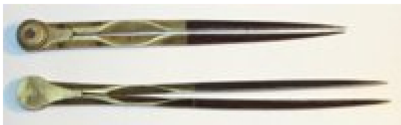


Figure 3.4. Two pairs of English eighteenth century dividers (~1740–1810) (collection Calculant)

early as 1589.[31] Dividers provided just part of the spectrum of mathematical tools that were under steady development. By the C18 such instruments were being produced in sets which included dividers, protractors, increasingly precisely scribed rulers, finer drawing pens, and other instruments, such as the proportional compass. The set of instruments, below in figure 3.5, is from ~1880.

One mathematician - Thomas Hood - a Doctor of Physic from Cambridge University, took as part of his teaching task the design and documentation of potentially useful

31. Daniel Speckle, *Military Architecture*, Strasburg, 1589 cited in John Robertson, *A Treatise of Mathematical Instruments*, Third Edition, 1775, reprinted Flower-de-Luce Books, Virgina, 2002, p. vii.

Figure 3.5. French drawing instruments ~1880 (including proportional dividers- top right) (collection Calculant)

instruments. One instrument which he describes emerged naturally from the concepts of dividers, and proportional scaling inherent in the proportional compass. This was the sector - a set of dividers, but with flattened legs upon which could be marked various scales.[32] It represented a considerable advance in aids to calculation.

The sector - an early calculating rule

Others (including Galileo Galilei) have claims to the invention of the sector.[33] Hood first described his sector in 1598. Also known as a proportional compass the sector consisted of two rulers of equal length joined by a hinge and inscribed with various scales, to facilitate, in particular multiplication and division (but which could also be used to assist in problems of proportion, trigonometry, and calculations of square roots). It utilises the geometric principle, articulated by Euclid, that the like sides of similar triangles are in the same proportion. By forming an equilateral triangle with

32. Taylor, *The Mathematical Practioners of Tudor & Stuart England*, p. 41.
33. for a picture of Galileo's made in about 1604, see `http://en.wikipedia.org/wiki/Sector_`, viewed 14 April 2012.

side and base in a particular ratio, multiplications in the same ratio could be read off for any other length of side allowed by the instrument.

Hood's sector was constructed by Charles Whitwell, a fine instrument maker.[34] However, the process of adoption was far from immediate. It was helped when, in 1607, Edmund Gunter, a young mathematician not long from his studies at Oxford, began to circulate his hand-written notes on *The Description and Use of the Sector* showing in particular how well it could be applied to the problems of navigation by Mercartor's charts. In this he found support from Henry Briggs. In 1597, Briggs had been appointed as the first Gresham Professor of Geometry at Cambridge University with an aim of bringing geometry to within reach of the people of London.[35] The very creation of this Chair indicated that there was a growing understanding of the potential utility of mathematical thinking. (However, the Chair was kept within the bounds of propriety by focussing on geometry which had less association with the "dark arts"). Briggs discharged his duty in popular education, amongst other things, by giving his lectures not only in the obligatory learned language of Latin, but repeating them in English in the afternoons. With Brigg's support Gunter's *Use of The Sector* was issued in print in 1623.

Sectors were in common use right through to the early twentieth century. Two sectors (from collection Calculant) are shown in table 3.2, *Two Sectors*, below. The first is a brass gunnery sector by Michael Butterfield, Paris. Michael Butterfield, and English clock maker was born in 1635 and worked in Paris from ~1680 to 1724. The second is an architect's sector by T. and H. Doublett who practiced their craft in London around 1830.

It is worth making a couple of observations about these sectors.

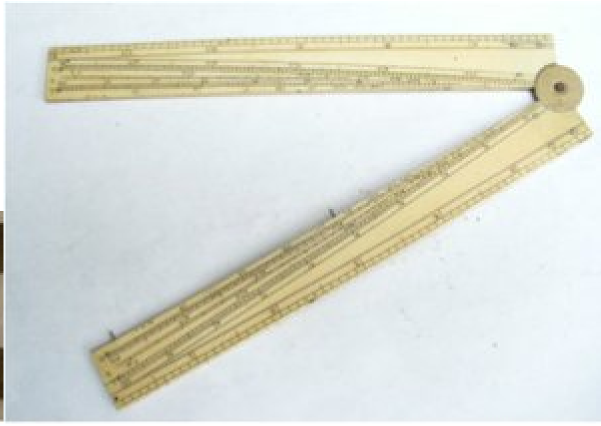
First, for reasons already mentioned, the learned designers of these early calculational aids seldom had the skills necessary to make them. Artisans, such as Michael Butterfield, who did have the highest relevant skills were frequently found amongst the members of the watch and clock makers guilds. Later specialist mathematical and scientific instrument makers (such as T. and H. Doublett) began to emerge.

Second, the construction of these aids required appropriate and available materials. For a gunner a robust brass sector made good practical sense. It needed to stay serviceable through the rigours of a battle. For an architect, 130 years later when sectors were being produced and used in larger number, the softer material of oxbone, which was both readily available and provided a white easily scribed ivory-like background for black engravings, created a much lighter instrument well suited to purpose.

The various scales which could be placed on sectors included trigonometric scales (e.g. sines and tangents) and linear scales for multiplication and division. To use these for multiplication and division one could, with a pair of dividers, set up a triangle of the required proportions.

34. Taylor, *The Mathematical Practioners of Tudor & Stuart England*, p. 41.
35. ibid, p. 54, 184

Table 3.2. Two Sectors



Brass French Gunnery Sector by Michael Butterfield ~1700 (collection Calculant)	Oxbone Architect's Sector by T. and H. Doublett ~1830 (collection Calculant)

John Robertson, in 1755, described the method thus: "To take a diftance between the points of the compaffes. Hold the compaffes upright, fet one point on one end of the diftance to be taken, there let it reft; and (as before fhewn) extend the other point to the other end."[36]

Using the same dividers, following the above, it was necessary to measure off the distance between the two "legs" of the sector at the required point along them.[37] As can be seen from the dividers in this collection, the accuracy of calculations using sectors was limited by the fineness with which their scales were rendered and the precision with which the points of the dividers could be applied to the task of measuring them. The process was thus slow, inherently inaccurate, and required considerable dexterity and practice to achieve a credible result.

Napier and the challenges of multiplication and division.

For astronomical, and many other calculations, the sector was never going to provide adequate accuracy. Yet the only way to do these better, absent great skill with an abacus, was by laborious long multiplication and division on paper. Only an elite in

36. John Robertson, *A Treatise of Mathematical Instruments As are usually put into a Portable Case*, Reprint of the Third Edition, 1775, Flower-de-Luce Books, The Invisible College Press, Virginia, USA, 2002. p. 6.
37. see for example, Edmund Stone, *The description, nature and general use, of the sector and plain-scale,: briefly and plainly laid down.*,Printed for Tho. Wright and sold by Tho. Heath mathematical instrument maker, next to the Fountain Tavern in the Strand., 1721, especially chapter IV, available from `http://books.google.com.au/books/about/The_description_nature_and_general_use_o.html?id=nqU2AAAAMAAJ&redir_esc=y`

any case had the mathematical literacy to carry such calculations out. Yet for those early mathematically literate people (for example, astronomers such as Kepler) the process was an enormously time consuming drudgery. There had to be a better way.

John Napier (1550–1617), Eighth Lord of Merchiston, was an imposing intellectual of his time. He pursued interests in astrology, theology, magic, physics and astronomy, methods of agriculture, and mathematics.[38] If these seem a strange set of areas, it is to be remembered, as Stephen Snobelen reminds us also of Newton, that few people now: "...have an understanding of what an intellectual cross-road the early modern period was. In fact, we now know that Newton was in many ways a Renaissance man, working in theology, prophecy and alchemy, as well as mathematics, optics and physics. In short, neither Napier nor Newton who came after him was "a scientist in the modern sense."[39]

Napier was an ardent Protestant. He wrote a stinging attack on the Papacy in what he would have regarded as his most important work.[40] It was a great success, and was translated into several languages by European reformers.[41] He also devoted considerable time to seeking to predict, on the basis of the *Book of Revelation* in the Bible, the likely timing of what was believed to be the coming apocalypse, which he concluded would come in 1688 or 1700. (At the time the "apocalypse" was not taken to mean so much the end of the world as the "temporary social disintegration and moral chaos, which is in turn mirrored in the devastation of nature.".[42]) In any case, Napier did not live long enough to be confronted with the prediction's failure. (Nor indeed did he need to confront the implications of Sir Isaac Newton's musings, in 1704, based on similar numerological investigation of the Bible, that the apocalypse would be no earlier than 2060).[43] Napier did however, live to see himself celebrated as the prodigiously gifted mathematician, and ingenious inventor, that he was.

In the course of his mathematical 'hobby' Napier invented an ingenious system of rods (now known as "Napier's Rods" or "Napier's Bones") which could be manipulated to enable two numbers to be multiplied together with little mental effort, although some manipulation of the rods. A drawing of a set from 1797 is shown in figure 3.6 below. The mathematical principle had been described by Al-Khwarizmi in the ninth century and had later been brought to Europe by Fibionacci. It involved using a lattice of numbers for multiplication (essentially a way of writing down a multiplication table). But by breaking the columns of the lattice into 10 rods sitting neatly on a board Napier created a calculational aid which was considerably easier (although still somewhat tedious) to use. Napier described this invention in his book *Rabdologiae* - a word he

38. Mark Napier, Esq, *Memoirs of John Napier of Merchiston: Lineage, Life and Times with a History of the Invention of Logarithms*, William Blackwood, Edinburgh, 1834.
39. Stephen D. Snobelen, "A time and times and the dividing of time": Isaac Newton, the Apocalypse and 2060 A.D., History of Science and Technology Programme, University of King's College, Halifax, undated, `http://www.isaac-newton.org/newton_2060.htm`, viewed 22 April 2012.
40. John Napier, *A Plaine Discovery of the Whole Revelation of St. John, 1593.*
41. William F. Hawkins, "The Mathematical Work of John Napier (1550–1617), *Bulletin of the Australian Mathematical Society*, Vol. 26, 1982, p. 455.
42. Snobelen, "A time and times".
43. ibid

created from the Greek for rod (rabdos) and calculation (logos) - which he published in 1617.[44]

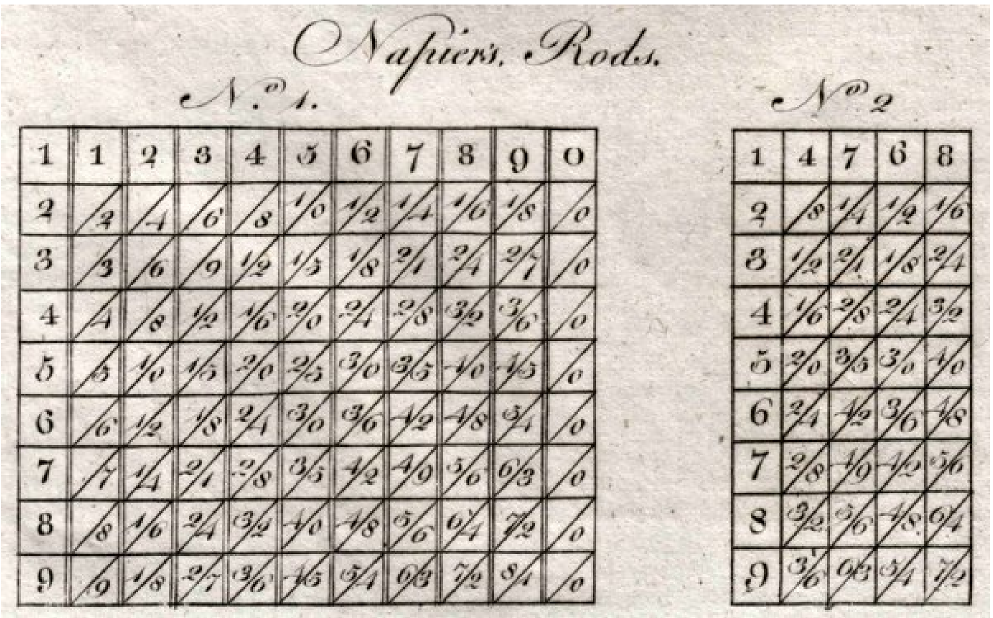


Figure 3.6. Depiction of Napier's Rods, 1797.[45] (collection Calculant)

The use of these rods can be illustrated by, say, multiplying 4 x 89. To do this read the 4 row of the last two rods (rod 8 and 9). Reading from right to left the right most digit in the result is 6 and 3 is carried. Moving left the next digit of the result is 2 to which the carried 3 must be added to give 5, with 3 to be carried. This then gave 356 as the result. This is no more than a simple way of replacing the multiplication tables that otherwise must be memorised. Pen and paper were still required especially if the multiplier was composed of more than one digit (e.g. 34 x 89), so that the process needed to be repeated for each of these (3×89 x10 + 4×89) then adding together these resulting "partial products".

More importantly, after 20 years of work Napier also devised and had printed in 1614, a set of tables, *Mirifici Logarithmorum Canonis Descriptio* ("Description of the Marvellous Rule of Logarithms"). These enabled direct multiplication to be carried out through simple addition. It is from these that modern tables of logarithms (with some additional conceptual improvements) are derived. These are based on the fact that the powers of numbers add when the numbers are multiplied (i.e. 2^3x2^4=2^7). This fact had been known since the time of Archimedes. But it was Napier who thought to use this sort of property to tabulate numbers in terms of the power of some "base" (2 in this example). In fact, Napier did not achieve his tables quite in this way. Rather it is believed he started from an earlier approach using a property from trigonometry.[46]

44. A copy of this book is available for download from this site's *e-library* (p 173).
46. namely sin(x)sin(y)=0.5{cos(x-y)-cos(x+y)} which as can be seen, also converts a multiplication into an addition of two terms (one negative).

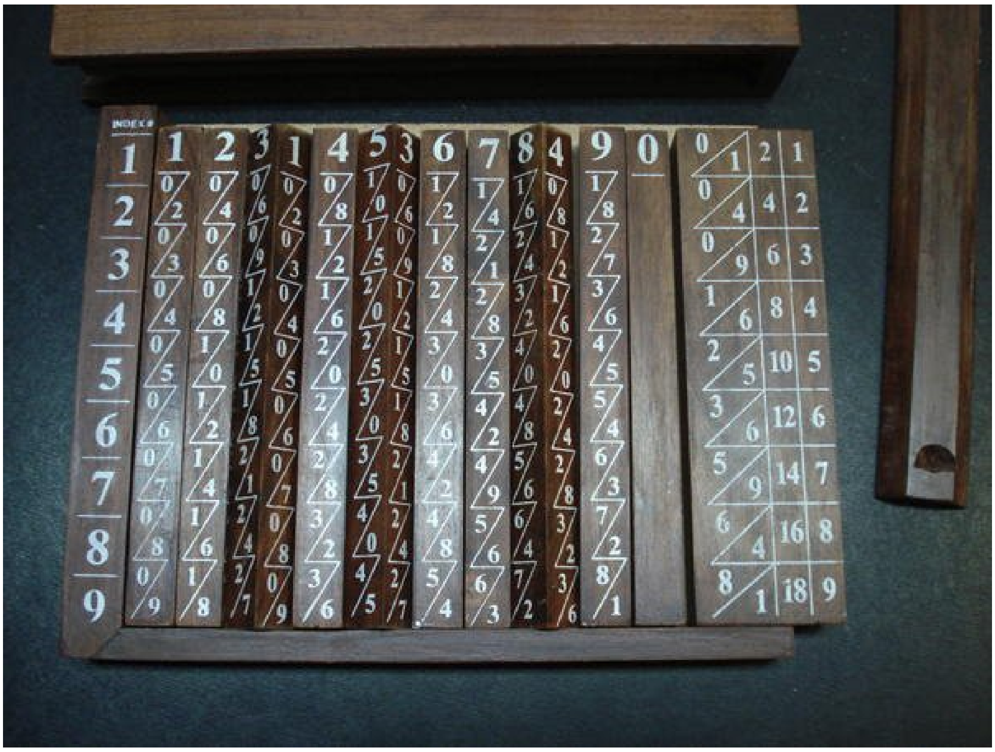


Figure 3.7. Set of Napier's Rods (of recent construction)
(collection Calculant)

This led him to a geometric argument based on the theory of proportions. Using this he constructed his functions.[47]

Napier's tables were welcomed by mathematically minded scholars across Europe. Indeed the astronomer Keppler (1571–1630) noted that his ground breaking calculations in relation to Tycho Brahe's astronomical observations would have been impossible without the use of the tables.[48]

Henry Briggs, the first Gresham Professor of Geometry at Cambridge (already mentioned) had travelled to meet Napier in Edinburgh in 1615. There it is reported the two men gazed in admiration at each other for a full quarter hour before finding words to speak.[49] Briggs had suggested in a prior letter that it would be good to develop tables of logarithms to base 10. Napier who had had a similar idea was unable to do the work because of ill-health. He was however very pleased that Briggs might carry the work through.

47. see for example, Denis Roegel, Napier's ideal construction of the logarithms, 12 November 2011, `http://locomat.loria.fr/napier/napier1619construction.pdf`, viewed 19 April 2012
48. Hawkins, "The Mathematical Work of John Napier, p. 456.
49. G A Gibson, Napier and the invention of logarithms, in E M Horsburgh (ed.), *Napier Tercentenary Celebration : Handbook of the exhibition*, Edinburgh, 1914, pp. 1–16, cited in J. J. O'Connor and E. F. Robertson, "John Napier", St. Andrews College, `http://www-history.mcs.st-andrews.ac.uk/Biographies/Napier.html`, viewed 14 Dec 2011.

Briggs did develop such tables of “common logarithms” the first of which gave the logarithms from 1 to 1000. It was published as a 16 page leaflet *Logarithmorum Chilias Prima* in 1617. His colleague Edmund Gunter at Gresham College published a more complete set from 1 to 20,000, in 1620. It was accurate to 14 decimal places.[50] The usefulness of such tables for serious calculations involving multiplication, division, and powers of numbers to high levels of accuracy was clear to those who became familiar with them. That knowledge and the tables themselves, usually supplemented by corresponding tables for trigonometric functions, spread rapidly.

In 1625 Wingate published a French edition of Brigg’s latest tables. A year later in 1626, Dennis Henrion published his tables “Traicté de logarithms” (of which there is a copy in this collection, see figure 3.8 below). Together with Wingate’s, these tables made available the power of logarithms across Europe.

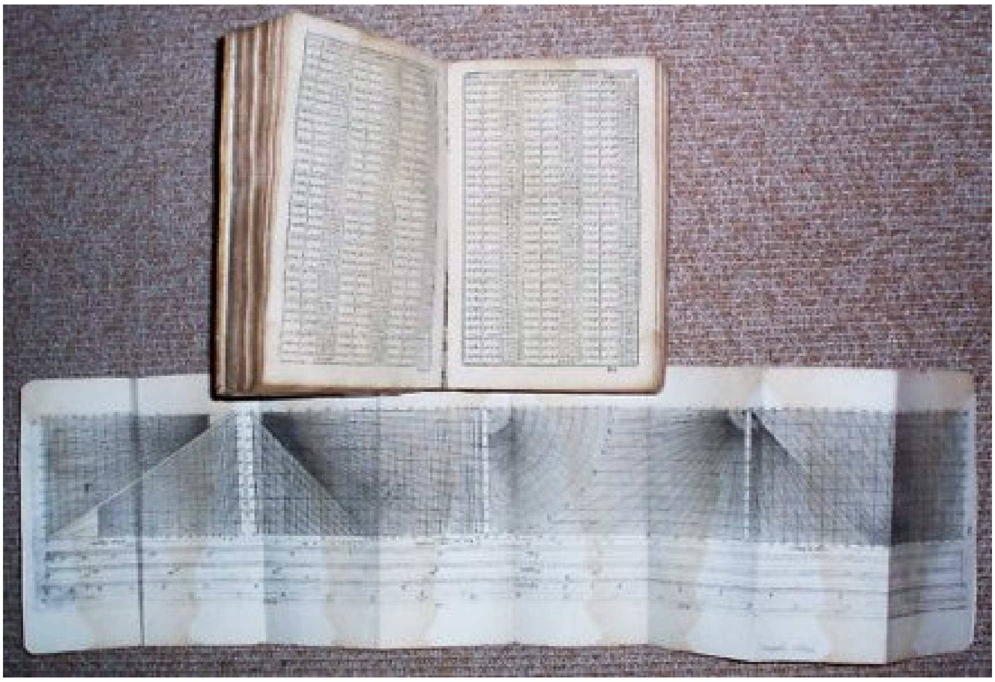


Figure 3.8. 1626: Traicté de logarithms by Dennis Henrion (collection Calculant)

Tables of logarithms, improved in various ways over time, were published either by themselves the *Tables* by Gardiner from 1783, or in the context of “ready reckoners” (e.g. Ropp’s *Ready Reckoner* from 1892, in this collection). Even up until the late C20 most school children were expected to have a passing understanding of how to use such logarithms prior to graduating to adult work. Extraordinarily useful as tables of logarithms were, a certain level of skill was required to use them. As many later in life would find, that skill also was not hard to forget. Tables of logarithms by Gardiner in 1783 are shown in figure 3.9 below.

50. ibid

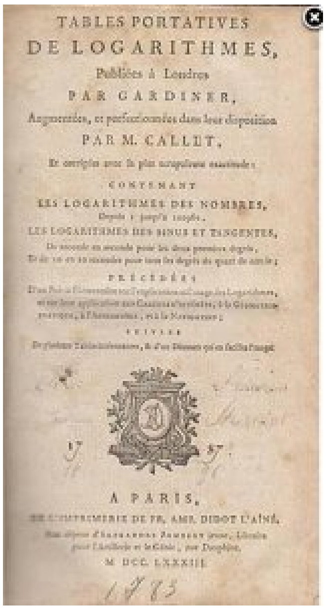


Figure 3.9. 1783: Tables Portatives de Logarithms by Gardiner (collection Calculant)

The process of multiplying with logarithms could only be as accurate as the accuracy to which they were tabulated. Using them required a certain level of meticulous writing down of intermediate numbers and careful addition and subtraction. Using them was not easy for many people, and certainly not quick for even more. At a time when the need for ready calculation was spreading in the economy there was a growing potential, if not yet realised, demand for other developments that would reduce the time, effort and skill required. Yet even in the C17 with an expanding interest in calculation there was a place for something that would be quicker and easier, even if not so accurate - somewhere between a quick rough calculation and the painstaking methodology of logarithms.

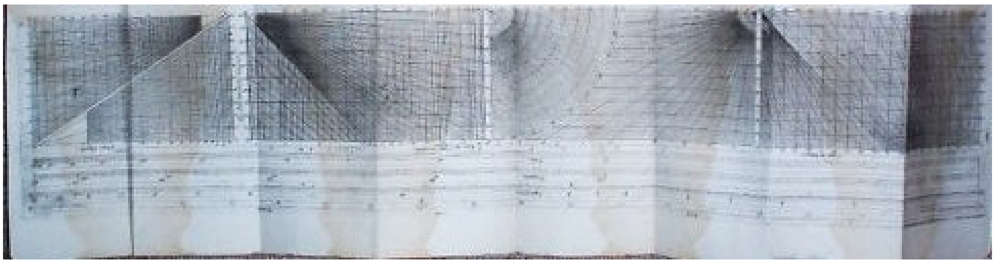
Proportional Rulers: The Gunter Scale

Not only had Brigg's colleague, Professor Edmund Gunter, published his *Canon triangulorum* in 1629, which contained logarithmic sines and tangents for every minute of arc in the quadrant to seven decimal places. In 1624 Gunter followed this with a collection of his mathematical works entitled *The description and use of sector, the cross-staffe, and other instruments for such as are studious of mathematical practise.* This work contained, amongst other things the detail of "Gunter's scale" (or "Gunter's rule") which was a logarithmically divided scale able to be used for multiplication and division by measuring off lengths. It was thus the predecessor to the slide rule.[51]

51. http://en.wikipedia.org/wiki/Edmund_Gunter - see also http://www.livres-rares.com/livres/HENRION_Denis-_Traicte_des_Logarithmes-95656.asp

In a second section (see below), the book details the design of a logarithmic proportional rule (derived from Gunter's 1624 publication), along with additional explanations, charts and other elaborations (see table 3.3, *Graphical Construction of Gunter Scales*, below). The proportional rule could be used directly by means of a pair of dividers to measure off lengths corresponding to logarithms and thus to evaluate multiplications and divisions.

Table 3.3. Graphical Construction of Gunter Scales



1624: Graphical Construction of Gunter Scales
reproduced by Henrion 1626
(collection Calculant)

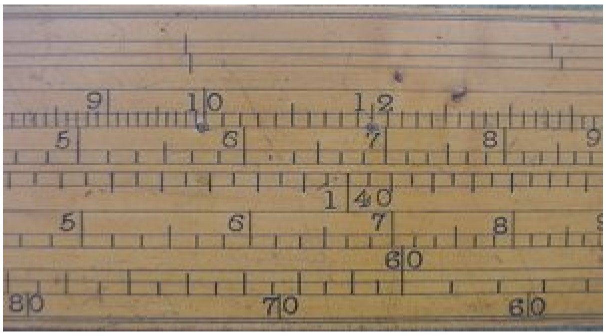
Gunter rules were usually equipped with both Gunter's combination of a logarithmic and a linear scale. Often they also included a range of other scales (notably trigonometric scales for navigational tasks, such as required to work from elapsed time, speed and changes to compass bearing to distances travelled). In addition, important constants could be marked on them as "gauge marks". Over the seventeenth century Gunter rules gained increasing acceptance and were used right through into the late nineteenth century. Some of the scales of a two foot long mid-nineteenth century navigational Gunter rule in this collection are shown in table 3.4, *The Gunter Rule*, below.

With its multiple scales (including the vital logarithmic scale) the Gunter rule was a lot more flexible in use than a Sector. It was much easier to make a rough calculation using it. All that was necessary was to measure off and add with dividers lengths against the various scales. This was much easier than having to write down the intermediate results. It was not long however before it was realised that instead of using dividers to add these lengths, the same thing could be achieved more easily by sliding two scales against each other.

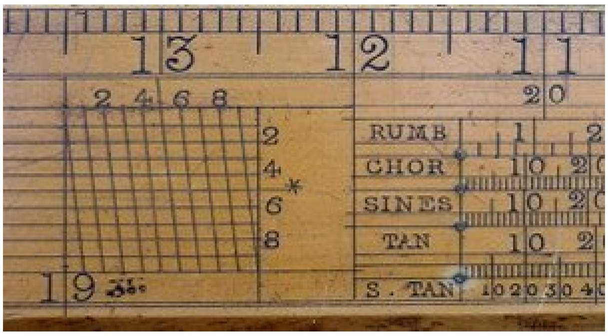
The evolution of the slide rule

The initial insight was that the logarithms of two numbers could be added by sliding two Gunter scales against each other. There is however debate about who was the first

Table 3.4. The Gunter Rule



Above: Logarithmic scales of Gunter rule



Above: Navigational scales of Gunter rule

Gunter Rule (1831–1843) by Belcher & Bros

(collection Calculant)

to realise this.[52] It was William Oughtred who published his design for a slide rule in 1632.

A series of designs followed. Three of these appear in Jacob Leupold's book *Theatrum Arithmetico-Geometricum*[53] of which Table XII (page 241), held in this collection, is shown in (i) in table 3.5, *Slide Rules*. (The top drawing is a design for a Gunter scale, and the two below are early designs for slide rules).[54]

In 1677 Henry Coggeshall desecribed a slide rule more like modern ones. Two rules with scales were held together with brass strips so one could slide past the other. This slide rule was found to have particular application for those who needed to do

52. See for example, Dieter von Jezierski, *Slide Rules: A Journey Through Three Centuries*, Astragal Press, New Jersey, USA, pp. 7–8, and Florian Cajori, *The History of the Logarithmic Rule*
53. A scanned version of this book may be downloaded from `http://ia600304.us.archive.org/33/items/theatrumarithmet00leup/theatrumarithmet00leup.pdf` (viewed 3 Jan 2012)
54. This page was held in a German family and placed on auction on *ebay* in January 2012 after the seller's grandfather, an art collector, who had held it and had it restored, passed away.

calculations quickly (and roughly) whilst on the job. In short it was a practical device for practical use.

One consequence of Britain's increasing strength in shipping and maritime trade was that trade became an obvious target for revenue raising. During the C17 taxation was aggressively applied to offshore trade. The income raised was in part invested in the increased naval capacity and colonial infrastructure required to protect shipping. Tax was applied to commodities as diverse as glass, paper, soap, vinegar, famously tea, and of course alcohol in wine, ale and spirits (the taxation of which began in 1643). One consequence of this was that the quantities of these in diverse containers needed to be audited.[55] This created a rapidly growing need for "gaugers" who could apply the mathematics of "stereometry" to estimating volumes of fluid held, and the corresponding alcoholic content. These measurements needed to be applied not only to barrels (whether on their side or standing), but also butts, pipes, tuns, firkins, puncheons and long-breakers (amongst other now long forgotten containers).[56] Given the lack of widespread mathematical literacy, it was essential to have aids to enable gaugers to do this. Extensive manuals, tables and guides were published, but even so, it was clear to practitioners that they really needed something easier to use.

In 1683 Thomas Everard , an English Excise Officer (who is credited with introducing the term "sliding rule"), began promoting a new 1 inch square cross section slide rule with several slides for calculating excise.[57] Shown in *(ii)* on the following page, below, is an English four sided Everard pattern sliding rule from 1759. It includes various gauging points and conversions to square and cube roots for calculating volumes.[58] In *(iii)* on the next page is a more modern looking slide rule shape, from 1821–84 by Joseph Long of London, also for use in gauging the amount of alcohol spirit in a container, and calculating the corresponding tax.

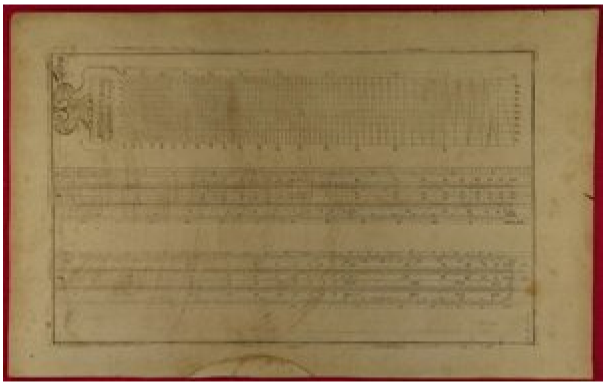
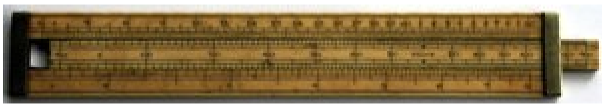
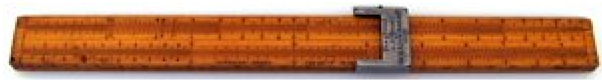
The slide rule evolved in what are early recognisable directions. The use of a hair line to read off multiple scales was suggested as early as 1675 by Sir Isaac Newton. Nevertheless, the introduction of a moveable cursor with this innovation included had to wait a century until a professor of mathematics, John Robertson, in 1775, added a mechanical cursor.[59]

It was Victor Mayer Amédée Mannheim, a Colonel in the French artillery and professor of geometry in Paris, who introduced the now familiar scale system, combined with a now fully functional cursor. This effectively brought together the key elements of what would become the modern slide-rule. He described it in a pamphlet published in 1851[60] An early slide rule made in 1893–98 by Tavernier-Gravet, but

55. Tom Wyman, "Kilderkins, Hogsheads & Dipping Rods: A Brief History of the Slide Rule", *Journal of the Oughtred Society*, Special Issue 2007, pp. 19–26.
56. ibid
57. Wyman, "Kilderkins, Hogsheads & Dipping Rods", p. 21.
58. The method of use is described in I. Rawbone, *The Royal Gauger*, Oxon, London, 1750
59. Dieter von Jezierski, *The History of the Slide Rule*, Abstract Press, New Jersey USA, 2000, p. 11.
60. A. Mannheim, *Règle à calculus modifiée*, Grande imprimerie Forezlenne, Septembre 1851. `http://www.linealis.org/IMG/pdf/Mannheim.pdf` viewed 24 June 2012; Jezierski, *The History of the Slide Rule*, p. 12.

based on a pattern devised by Lenoir in 1814 *(iv)* below is also shown, now fitted with a brass cursor. In *(v)* below is a slide rule from about 1928 by the firm Keuffel and Esser with cursor and familiar scales. The Faber Castell 2/83N Novo Duplex slide rule *(xii)* on the facing page with its multiple scales on both sides is often spoken of as the high point reached in elegance and utility of the straight slide rules. Production of it lasted until 1973 when electronic calculators rendered it obsolete. A progression of such slide rule designs is shown in table 3.5, *Slide Rules*, below.

Table 3.5. Slide Rules

Note	Date	Maker	
(i)	1626–1726	Jacob Leupold 3 designs	
(ii)	1759–69	Edward Roberts Everard pattern gauger's rule	
(iii)	1821–84	J. Long Alcohol gauger's rule	
(iv)	1893–98	Tavernier Gravet Slide rule	
(v)	~1928	K&E Slide rule	

Continues...

(xii)	1967–73	Faber Castell 2/83N Novo Duplex	

(All the above are from collection Calculant)

Over the late nineteenth and first half of the twentieth century a two-way innovation race began to achieve accuracy and capacity on the one hand, and compactness and versatility on the other. Greater accuracy required longer slide rules so they could be more finely divided. To overcome the practical difficulties American bridge engineer Edwin Thacher effectively chopped two 10 metre slide rules into 20 segments and set them side by side around in a cylindrical manner to form an open "cage" with forty scales and an equivalent length of 20 metres. The cage of scales could rotate around a fixed inner cylinder which bore corresponding scales. Compacted in this way the instrument *(vii)* on the next page, which was patented in 1881, remained very bulky, but capable of multiplications and divisions to an accuracy of 4 to 5 decimal places. Professor Fuller subsequently developed a cylindrical slide rule *(viii)* on the following page which wrapped the scale around the cylinder in a spiral pattern giving a more compact instrument with a scale of equivalent length 12.7 metres, able to calculate to 3 to 4 decimal places. This was sufficiently practical to go into widespread use.

The search for compactness and versatility (at the cost of accuracy) continued resulting in a wide variety of circular pocket slide rules. Examples here are the the very first circular slide rule to be introduced in America, Palmer's Computing Scale *(vi)* on the next page, the Supremathic *(ix)* on page 87, and the Fowler *(x)* on page 87 - which combined a pocket watch style system with multiple scales. The Otis King pocket cylindrical slide rule *(xi)* on page 87 was a particularly charming slide rule with its neat spiral scale design but of course less accurate than its bulkier predecessors, being equivalent to 1.7 metres in length. These are shown in table 3.6, *Circular Scales*, below.

Table 3.6. Circular Scales

Note	Date	Maker

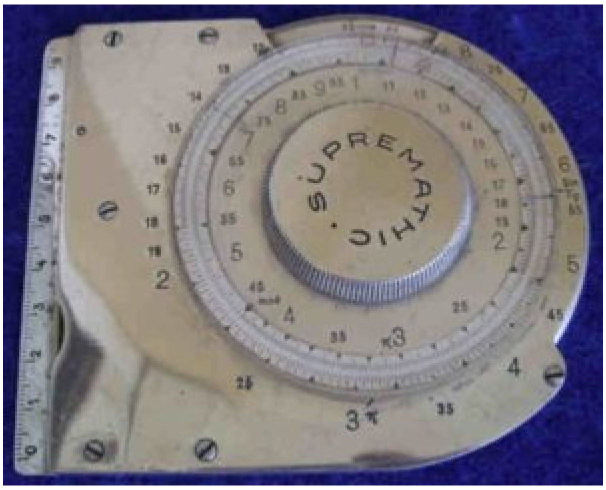
Continues. . .

(vi) 1847 Palmer Palmer's Computing Scale

(vii) 1911 K&E Thacher's Calculating Instrument

(viii) 1926 Prof Fuller's Cylindrical Slide Rule Model 2

Continues. . .



(ix) 1935 Suprematic Circular Slide Rule

(x) 1948 Fowler Jubillee Magnum

(xi) 1960 Otis King Model LC

Continues. . .

(All the above are from collection Calculant)

The above is notable for the extent to which the slide rule, in its multiple variants was able to be shaped into a tool of trade in multiple emerging and growing professions. Its advantage over logarithm tables was its speed of use at the expense of complete accuracy. As noted above, where equivalent accuracy was created the instrument became very large and clumsy.

Nomographs

We may add two further considerations to that of accuracy, and that is skill and speed. The slide rule was well designed for a professional, such as an engineer, who might have both facility in logarithms and the capacity to understand and evaluate equations. However, the increasing complexity of production called for employees with multiple skills and knowledge bases. Not all of these could be expected to be able to carry out necessary calculations from first principles. One approach to enabling them nevertheless to proceed was to provide tables of results. "Ready Reckoners" provided this sort of facility giving, for example, interest tables for the calculation of mortgages.

For more complex equations with multiple variables it was either train workers to evaluate the equations from first principles or find some other way of solving them. Nomography provided an approach to achieving this. This methodology was invented by Maurice d'Ocagne (1862–1938) in 1880 and was used through to the 1970s. Its principles are described in several good references.[61] A simple example of the approach is given in figure 3.10 below:

This simple nomograph is used to add two numbers (one in column a) and the other (in column c).[62] The properties of similar triangles give the result that a line drawn between the two numbers will cut the middle column (b) at the required sum. Since the outer scales may be logarithmic or trigonometric many more complicated expressions are able to be evaluated using this sort of approach.

61. eg. Ron Doerfler, "The Lost Art of Nomography", *The UMAP Journal*, Vol 30, No. 4, 2009, pp. 457–94; downloadable from `http://myreckonings.com/wordpress/wp-content/uploads/JournalArticle/The_Lost_Art_of_Nomography.pdf` viewed 11 July 2012 elegantly outlines the basic principles. Carl Runge, *Graphical Methods*, Columbia University Press, New York, 1912; readable online at `http://www.archive.org/stream/graphmethods00rungrich#page/n0/mode/2up` viewed 11 July 2012 provides an excellent overview of the relevant graphical methods for solving equations of many sorts.
62. A similar example is given in Raymond D. Douglas and Douglas P. Adams, *Elements of Nomography*, McGraw Hill, New York, 1947, p. 30; cited in David D. McFarland, "Addition and Subtraction With Slide Rules and Allied Instruments Part I, *Journal of the Oughtred Society*, Vol. 12, No. 2, Fall 2003, p. 34.

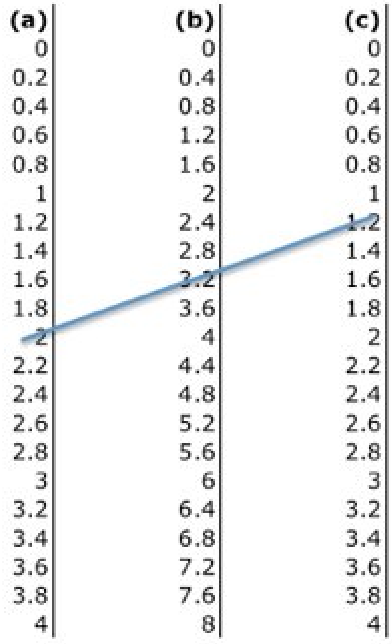


Figure 3.10. A simple nomograph for calculating the sum of two numbers (b=a+c)

Many nomographs were produced. Many were just printed on card allowing calculations to be read off using a rule as above. They were mostly if not invariably designed for a particular purpose.

It was also possible to create mechanical nomographs in which the scales were laid out and able to be read by turning pointers. Two devices which utilise these nomographic principles are shown below. The first (figure 3.11) is a Bloch Schnellkalulator from ~1924. The second (figure 3.12) is a Zeitermittler from ~1947. In both cases these are nomographic devices for calculating parameters required to cut metal with a lathe. The Bloch Schnellkalulator is notable for its use of a linked mechanism which enables the results of one calculation to be fed as the input to another.

Some Reflections

As the above suggests, multiple solutions were emerging to multiple formulations of the "problem of multiplication". Each solution had its limitations, whether it be ease of use in different practical circumstances, or accuracy. Thus, although slide rules had apparent advantages over Gunter rules, and Gunter rules over sectors, none of these simply vanished once the other had been invented. As Robertson noted in 1775, Gunter rules had simply been added to sectors as available approaches.[63] Indeed, as shown by the objects in this collection, Gunter rules and sectors continued to

63. Robertson, *A Treatise of Mathematical Instruments*, 1775.

Figure 3.11. Bloch Schnellkalulator~1924
(collection Calculant)

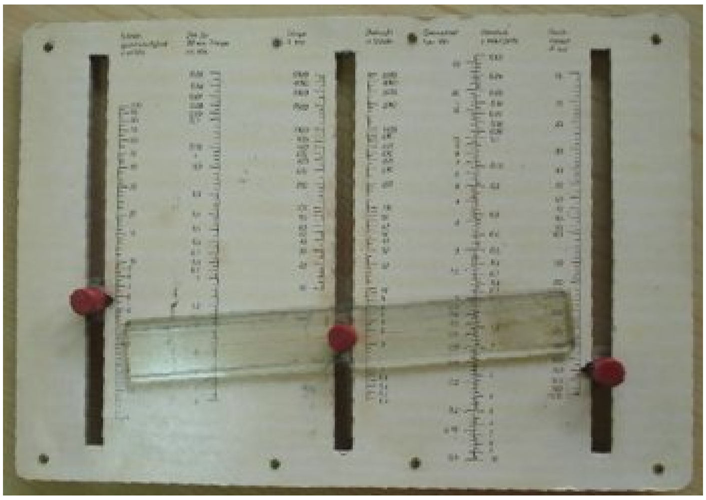


Figure 3.12. Der Zeitermittler~1947
(collection Calculant)

be used right up into the nineteenth century. Even to this day, nomographs, often represented now in the form of computer graphics, continue to be used for particular applications.,[64]

This undermines the simple minded view of innovation as a linear process of invention and improvement. Rather, as in any transitional period, multiple strands of change were in motion. They were deflected or shaped along the way by different motivations and pressures. One of these pressures was simply intellectual conservatism. That included the usual suspicion of practical compromises by those privileged to be able to

64. eg for brewing: *John Palmer, How to Brew, Chapter 15*[65] viewed 16 July 2012; or to estimate boiling temperatures at various temperatures *Pressure Temperature Nomograph*,[66] viewed 16 July 2012

focus purely on intellectual pursuits. When Gunter applied for a chair in mathematics at Oxford he was rejected by the Warden of Merton College, Sir Henry Saville. The reason given was that his instruments were “mere tricks”.[67] Rejected from Oxford Gunter took up the Gresham Chair at Cambridge University. As noted earlier, this position was dedicated to exposing mariners and others to the potential usefulness of mathematics. Somewhat later an Experimental Philosophy group would grow up at Oxford (and in 1672 would gain a Royal Charter from King Charles II and become the Royal Society).[68]

There is a range of other factors that would have affected the spread and adoption of a new instrument such as the slide rule. For example, in order to manufacture slide rules for mass distribution, the skills and techniques for scribing logarithmic scales would need to be picked up by potential instrument makers. Until many were produced and means were found to cheapen the process, costs of new instruments might well have been at a premium. Perhaps more important would have been the need to learn how to use them. The naval profession was not considered a place for scholars. Rather training was on the job and at the hands of senior sailors and officers. In short, whether in marine environments, or on land amongst architects, builders and planners, skills deemed necessary for doing the job were taught from master to apprentice in the age old fashion of the guilds. This was a very suitable way of passing on stable and established best practice. It was not necessarily so receptive to or good at transmitting new fangled ideas of scholarly gentlemen living and working in the privileged seclusion of universities. The Gresham Chair was intended to break through that, but it was too large a job to be achieved by any one such establishment.

The use of scaled functions to calculate (whether using principles of similar triangles, trigonometric relationships, or logarithmic properties), whether embodied in tables, or whether incorporated into instruments (such as sector, Gunter scale, or slide rule) had one key limitation. Apart from the logarithmic scale, the remaining scales or marks were generally about solving specific problems. These were problems such as how to multiply or divide by physical constants or how to work out trigonometric relationships and apply them to various problems. Almost all of the calculational aids available were limited in accuracy either by the scales used or number of decimal places to which tables could be listed. They did not provide in any useful way for addition and subtraction which many found challenging to do in their heads. And being focussed on specific problems, these aids lacked the generality (as well as accuracy) that might be required to solve the multiple diversifying calculational tasks of increasingly complex societies.

That there was a need for something new is clear with the clarity of hindsight, especially centuries later. But to cast the track of innovation simply as fulfilling an obvious need is too simplistic. As we have noted already, there were at least two (and probably many more) publics for whom innovation in mathematics might have

67. Robert Wilson, “Who invented the calculus? - and other 17th century topics”, Gresham Lecture, Gresham College, Cambridge, 16 November 2005. `http://www.gresham.ac.uk/sites/default/files/calculus1.pdf`, viewed 2 May 2012.
68. ibid

relevance. But that does not mean that they recognised that relevance. And in any case mathematical inventions might have very different relevance to those publics. The two publics we have already identified were:

(i) those intellectuals (whether labelling themselves as philosophers, mathematicians or some other way) focussing on mathematical exploration, and those other natural philosophers including the emerging group of “experimental philosophers” who might utilise their work. For these there might be the delight of embodying mathematical ideas in devices, or in the case of what would become later known as scientists (for example, astronomers) the prospect of doing away with the tedium and delay of endless simple mathematical calculation.

(ii) the practitioners of practical arts - whether sailor, cartographer, or clerk who might appreciate a tool that would ease their work. Complementing this there was a slowly growing demand for larger numbers of “calculators” - that is, people who could calculate. Given that this was not a widespread skill, as already noted, anything that might ease the learning and teaching of the skills, or replace the need for it with some device, could over time prove attractive.

As Laird and Roux note, “The establishment of mechanical philosophy in the 17th century was a slow and complex process, profoundly upsetting the traditional boundaries of knowledge. It encompassed changing view on the scope and nature of natural philosophy, an appreciation of “vulgar” mechanical knowledge and skills, and the gradual replacement of a casual physics with mathematical explanations.”[69] It was this slow and complex process that would eventually give rise to mechanised calculation. But as suggested above, the developments would be complex, many stranded, and over time increasingly tangle together the two worlds of the philosophical and practical arts.

3.1.4 Mechanical Calculation - first steps

As with much of mathematics, the interest in mechanisation of calculation was to some extent a re-discovery of similar interest several thousand years before. The Antikythera mechanism originating in ancient Greece has already been mentioned. It was, in a sense, a hand cranked calculator for predicting regular events such as celestial formations and important occasions of state. In ancient Rome, there had also been considerable use made of pumps, levers, wheels and gears, for a variety of uses in construction, and destruction - especially in the use of machines of war.

The Roman emphasis on the importance of transport for the state had led also to the use of measuring machines including the odometer of Heron of Alexandria - a device used to measure distance traversed by a rotating carriage wheel by coupling it to a

69. W.R. Laird and S. Roux (eds.), *Mechanics and Natural Philosophy before the Scientific Revolution*, Springer, US, 2008, p. 259.

system of incrementing gears. As described by Vitruvius (~15 BC),[70] the rotation of the carriage wheel was measured by this device in units, tens, hundreds, thousands and tens of thousands of paces.[72] This sort of counting mechanism could form a natural basis for the important "carry" mechanism (e.g. where 9+1 is converted to 10) of an adding machine. Still, for that potential to be capitalised upon there had to be a recollection or re-discovery of the sort of mechanism, and perhaps more importantly, an interest in constructing one. As already mentioned, the budding interest in mechanisation in the early C17 (building in particular on the widely understood appeal of the success of clocks, clockwork, and new mechanical insight about nature) provided an appropriately fertile base for that interest.

Given the increasingly multi-stranded stress on the importance of calculation, it is not surprising that at least some natural philosophers in the England and Europe took up the challenge of how best to facilitate it. Even though often distant from mundane economic or practical need there were intellectuals of the day who shared not only an enthusiasm for discovery, but also a growing enthusiasm for invention. It was only a matter of time before a growing interest in mechanisation would intersect with desire to simplify the process of calculating solutions to a variety of mathematical problems.

One group of such intellectuals were the Jesuit theologians who were now emerging as mathematical thinkers. By 1650 some 50 mathematical chairs had emerged in Jesuit colleges across Europe.[73] It was Joannes Ciermans (1602–1648), a Flemish Jesuit, who in 1641 published one of the most comprehensive surviving courses covering geometry, arithmetic and optics. These were presented in a practical way, designed for his students who were mostly expected to become military officers.

In the "Problemata" for one week of his course, Ciermans noted slightly obscurely (loosely translated) that while many seek savings in multiplying and dividing the outcomes usually require more effort to do so than from first principles. However, he wrote, there is a method with "rotuli" (normally rolls of parchment for writing upon, but could also intend the more modern diminutive for rota i.e. little wheels) with indicators (or pointers), which enables multiplication and division to be done "with a little twist" so the work is shown without error.[74]

It is not clear if the device described by Ciermans existed, was envisaged, or was

70. *Vitruvius, De architectura ("Ten Books on Architecture (trs Morris Hickey Morgan), Book X, Chapter IX.* ^[71] viewed 18 July 2012

72. Flavio Russo, Cesare Rossi and Marco Ceccarelli, "Devices for Distance and Time Measurement at the Time of the Roman Empire", in Hong-Sen Yan and Marco Ceccarelli (eds.), *International Symposium on History of Machines and Mechanisms*, Proceedings of HMM 2008, Springer, US, 2009, pp. 102–6.

73. ibid p. 260.

74. P. Ioanne Ciermans, Mat Professsore, *Annus Positionum Mathematicarum Quas defendit ac demonstrauit*, Soctis Jesu,1641, Novembris Hebdomas, Prima Problemata. This section reads, *inter alia*: "PROBLEMATA Multiplicandi, diuidendique numeros, compendia quaesiuere multi, & inuenere, sed plus fere, sua instrumenta ut concinnent, absumunt temporis, quam communi modo numeros permiscendi exigeret labor. Nos itaque ita paruam rotulis instruimus machinam, ut indiculis tantum nonnihil contortis opus sit, ut propositu quemcunq; per datum numerum, multiplicemus, partiamurque, idque sine ulla quidem erroris suspicione, tam certo ordine movenutur haec omnia, numerumque multiplicatum, aut divisum exhibit."

merely suggested. It is certainly not clear precisely how it would have worked. One could speculate, since Ciermans refers to both logarithms and rabdologiae, that it might have embodied some form of Napier's rods on rolls. But it could have involved little wheels. It could just have been a way of checking accuracy by displaying the progress in a calculation using parchment rolls which progressively revealed each intermediate step. Indeed an improvement to method of this sort might well have provided greater improvement in arithmetic speed and accuracy than some more complicated mechanical approach. Tellingly this demonstrates how much may have been lost from the highly perishable historical record over four hundred years. Indeed, it was only in the last fifty years of the twentieth century that more tangible evidence did emerge that a quarter of a century before Cierman's remarks a calculating machine had indeed been constructed using Napier's rods set in rollers together with a mechanism for adding and subtracting based on the interaction of little wheels.

Schickard's Calculating Clock

It is hardly surprising that the first steps towards innovation in the mechanisation of calculation would come from either someone who was from the nobility, or could draw on noble patronage, and with an established background (in commerce, church or state). This was practically a prerequisite to having had access to the education, adequate time and resources sufficient to enable such ideas to be worked out, and then implemented by employing the guild skills of clock makers or other skilled artisans.

The earliest of the known Modern attempts at mechanising calculation is that of Wilhelm Schickard (1592–1635). He was born in the German town of Herrenberg, near Tübingen, gained his first degree in 1609 and a Master degree in theology in 1611, and became a Lutheran minister in 1619. After a spell as professor of Hebrew at the University of Tübingen, in 1631 he was appointed to the University as professor of astronomy, mathematics and geodesy.[75] Importantly he was an intellectual who had grasped the importance of applying his intellect beyond theory to practical matters. He was an accomplished engraver and cartographer and devoted considerable time to the first geodetic land survey of Württemberg.[76]

It is worth remembering the smallness of the circle of people who had the necessary skills, interests and resources to participate in what we now know of as science. The astronomer Johannes Kepler, who was instrumental in demonstrating that the planets might most simply be considered to move in ellipses around the sun, and

75. "History of Computers and Computing, Mechanical calculators, Pioneers, Wilhelm Schickard", http://history-computer.com/MechanicalCalculators/Pioneers/Schickard.html viewed 7 May 2012

76. Benjamin Nill, "WWW-basierte interaktive Visualisierung der Rechenmaschine Wilhelm Schickards durch ein Java 3D-Applet", Studienarbeit von Benjamin Nill, Betreuer: Dr. Bernhard Eberhardt, Frank Hanisch, Wilhelm-Schickard-Institut für Informatik Graphisch-Interaktive Systeme (GRIS), Universität Tübingen, September 1999
http://www.gris.uni-tuebingen.de/edu/projects/schickard/studw_1.pdf viewed 21 June 2012.

who provided other crucial insights that would later inform the work of Newton, was a graduate of the University of Tübingen where he had studied under Magister Michael Maestlin, one of the leading astronomers of the time.[77] It was through Maestlin that Kepler came to know of the still suppressed work of Copernicus for who's solar-centric theory he became a strong supporter.[78] His first book, *Mysterium Cosmographicum*, was published in Tübingen and it was presumably on one of his visits to Maestlin there that he was introduced to Schickard. They had much in common - both being mathematically gifted and knowledgeable, and both also sharing strong Lutheran theological interests.[79]

Schickard's invention is revealed in two letters to Kepler, the first being in 1623, where he describes an "arithmeticum organum" ("arithmetical instrument") that he has invented, but later as a Rechen Uhr (calculating clock) in a surviving note to the artisan constructing his machine.[80] It is not known whether a complete Schickard's "calculating clock" was ever actually built, although given he describes it operating, a prototype of the lower (adding machine) part was almost certainly built, and very probably a prototype of the whole was commissioned since he also ordered one for Kepler. Our knowledge is based on Schickard's notes to artisans on building the machine, and drawings (see below) as well as comments to Kepler (discussed later). However, in his second letter Schickard made clear that he was not giving a full description.[81] Surviving sketches are shown in table 3.7, *Sketches by Schickard*.

The first replica was of Schickard's device was constructed in 1960 by Professor Bruno, Baron von Freytag-Löringhoff of the University of Tübingen. The figure 3.13 in this collection, shown below is more recent.[86]

Through twenty-first century eyes the principle of the machine was simple enough. It was primarily intended for addition, subtraction and multiplication. (Division was possible but difficult with this device).

The bottom of the machine was for adding and was itself a real innovation. All mechanical adding devices work by moving some object in proportion to the amount to be added. The simplest adding device is a "ruler" whose numbers are laid out uniformly along it. Two different distances corresponding to two different numbers can be added together and read off. Schickard utilised successive rotations of a wheel

77. Max Caspar, *Kepler*, translated and edited by C. Doris Hellman, Dover Publications, London, 1993, p. 48.
78. ibid p. 46.
79. ibid p. 49; "History of Computers and Computing, Mechanical calculators, Pioneers, Wilhelm Schickard"
80. For these letters see Schickard's complete correspondence in Friedrich Seek (ed)., *Wilhelm Schickard Briefwechsel*, Frommann-Holzboog, Stuttgart-Bad Cannstatt, 2002, i:135 and 141–142.
81. "â�¦Arithmeticum organum alias delineabo accuratius, nunc et festinate hoc have", which translates as ". . . I will describe the computer more precisely some other time, now I donâ��t have enough time" - see also `http://metastudies.net/pmwiki/pmwiki.php?n=Site.KeplersLetters`
86. This fully working replica (albeit with the limitations to the carry mechanism as described below) was constructed through a collaboration of three European public museums and one private museum, under the coordination of Reinhold Rehbein. It was purchased for collection Calculant in 2011.

Table 3.7. Sketches by Schickard

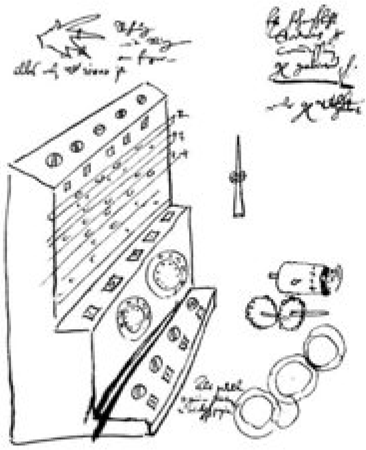
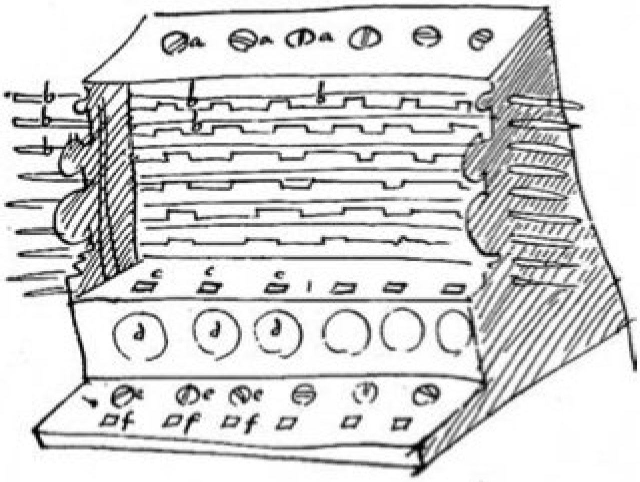
	
Original sketch by Schickard ~1623[84]	Second sketch by Schickard[85]

Figure 3.13. Working replica of Schickard's Calculating Clock (1623) (collection Calculant)

to add numbers, and perform the carry (9+1=10) in a manner reminiscent of an ancient Roman odometer.

Figure 3.14. Schickard front panel
(collection Calculant)

As can be seen in the replica above, a line of disks represented successive places (from the right units, tens, hundreds . . .). Behind the disk a gear wheel was turned which, when it passed from "9" to "0" engaged with the wheel to the left to move it by one unit. Adding was achieved by anti-clockwise rotations, subtraction by clockwise rotations. The set of knobs in the base allowed intermediate results to be recorded.

The vertical section at the top was a mechanical cylindrical embodiment of Napier's bones (published six years earlier) to aid multiplication.

> It worked like this: Consider 35 x 498. [The calculations is actually performed as (30+5) x (400+90+8).] The multiplicand 498 is set using the knobs along the top of the machine which rotates the vertical cylinders to show a number from 0–9 in the top "1" row of windows. Using these knobs, 498 is is set along row "1" starting with 8 on the right. Then the windows in the row for 5 are opened by pulling its shutter to the right (then displaying the numbers (20 45 40). [This really represents 2000+450+40 or 5 x (400+90+8).] These "partial products" are then added up using the corresponding disks of the adding machine in the base (which from the right represent the accumulated numbers of units, tens, hundreds . . . etc), and this is then repeated for the next digit (3) of the multiplier [that is, 30 as above - therefore starting from the second disk from the right] giving the final result 17430.[87]

At least as interesting as the specifics is the mood of the moment. Schickard was a member of a small network of people spanning Europe and the UK (Napier in Scotland, Kepler and Schickard in what is now part of Germany) of somewhat different backgrounds. All were either resourced by inherited or institutional wealth (Napier), Kepler (under patronage of Emperor Rudoph II via Tyco Brahe) and Schickard (supported by the University). All were well educated in the classics and inspired by multiple and to an extent overlapping passions of theology, philosophy

87. This particular calculation is explained in more detail in Friedrich W. Kisterman, "How to use the Schickard calculator", *IEEE Annals of the History of Computing*, January–March 2001, pp. 80–85.

Figure 3.15. Schickard rear view showing Napier cylinders
(collection Calculant)

(including natural philosophy) and mathematics. And all now shared a sense of excitement that it might be possible to make break-throughs in systematic knowledge. The reward could include the pleasure of aiding each other to break new intellectual ground coupled with the pleasure of achievement, the glow of approval from each other, and perhaps not only the admiration but the possibility of continued or enhanced support from patrons.

The first European 'scientific journal' (*Le Journal des sçavans* - later *Le Journal des savants*) published its first edition on 5 January 1665. Once established, in 1666, it soon became the written forum for the Paris Academy of Sciences.[88] (In England, the *Philosophical Transactions of the Royal Society* began publishing only three months later in March 1666.) Prior to that (indeed after, in many places) news of discoveries was spread by letters sent to trusted figures in the small 'invisible college' of thinkers. Thus, for example, William Oughtred (first to publish about the sliding part of the slide rule) was one of the key contact points in England. Others would learn of developments in the popular seminars he delivered at his home.[89] The network was thus characterised by a shared excitement and objectives, a developing communication system, the stimulus of possible positive outcomes, and consequently

88. Henry Hallam, *Introduction to the Literature of Europe in the Fifteenth, Sixteenth, and Seventeenth Centuries*, Harper & Brothers, 1842, p. 406.
89. Michael Williams, *A History of Computing Technology*, p. 107.

a level of competition (as revealed in the often quite tense and hard fought battles beginning to rage over who had primacy in any particular insight or invention).

Kepler was greatly excited by the publication of logarithms which he utilised to greatly reduce the enormous calculational effort in his computations of *The Rudolphine Tables* and later works. He dedicated his *Ephemerides* to Napier in 1617.[90] Professor Briggs (astronomer and geometrician) now at Gresham College, Cambridge, as mentioned earlier, was a close colleague of Napier and had written of his tables "...I never saw a book which pleased me better or made me more wonder".[91] He would later take Napier's work further forward producing new tables of logarithms to base 10. In 1609 Briggs was also impatiently awaiting Kepler's exposition on ellipses.[92]

Finally, of Schickard, Kepler wrote admiringly that he has "a fine mind and is a great friend of mathematics; ... he is a very diligent mechanic and at the same time an expert on oriental languages."[93] It is known that Kepler and Schickard had discussed applications to astronomical calculation by Kepler of Napier's logarithms and rods as early as 1617. This may well have inspired Schickard to find a mechanical embodiment of the rods.

On 20 September 1623 Schickard wrote to Kepler to tell him that:

> What you have done in a logistical way (i.e. by calculation) I have just tried to do by mechanics. I have constructed a machine consisting of eleven complete and six incomplete ("mutliated") sprocket wheels which can calculate. You would burst out laughing if you were present to see how it carries by itself from one column of tens to the next or borrows from them during subtraction.[94]

In a second letter to Kepler, on 25 February 1624, Schickard notes that he had placed an order for Kepler for a machine. He notes that unfortunately when half finished the machine had fallen victim to a fire and the mechanic did not have time to produce a replacement soon.[95]

The most often noted deficiency of Schickard's machine was in its "carry mechanism" (the mechanism that moves the displayed result from 09 to 10 when 1 is added). In Schickard's design it seems that the "carry" was achieved every time an accumulator wheel rotated through a complete turn, by a single tooth catching on an intermediate wheel causing the next highest digit in the accumulator to be increased by one.

90. Mark Napier, *Memoirs of John Napier* pp. 416, 432.
91. O'Connor andRobertson, "John Napier", `http://www-history.mcs.st-andrews.ac.uk/Biographies/Napier.html`.
92. Mark Napier, *Memoirs of John Napier*, p. 421.
93. Quoted in Herman Goldstine, *The Computer from Pascal to Von Neuman* , Princeton University Press, Princeton, p. 6.
94. Translation as quoted in Michael Williams, *A History of Computing Technology*, 2nd Edition, IEEE Computer Society and The Institute of Electrical and Electronics Engineers, Inc., USA, 1997, pp. 120–1.
95. ibid

Carrying a number required extra rotational force to be applied (since more than one wheel had to be moved simultaneously). If, for example, 1 was to be added to a number like 99999 to give 100000, all of the wheels bearing a 9 would have to be moved by the force applied to the right hand wheel as it moved from 9 to 0. Unfortunately that would require so much force as to break the machine. In short, the adding part of the machine would jam if too many numbers had to be carried simultaneously.

In addition, the machine was not simple to use. The use of Napier's rods to effect multiplication required considerable arithmetic knowledge - quite probably so much that the problem would more easily be done with pen and paper, or jetons and counting board. Probably Schickard's idea never moved beyond the prototype stage. Nevertheless, it was an inventive start. And the dynamic of the times, which had led to it being conceived went beyond the circumstances of Schickard. A skein of motivations had contributed to it being a potentially rewarding moment for Schickard to explore the ways a "clock" might calculate. Regrettably, Wilhelm Schickard, his family, and thus his calculating clock, all fell victim to the plague that followed the Thirty Year war and his work slipped into obscurity for three hundred years.

At the heart of Schickard's invention had been the idea of combining a convenient embodiment of the multiplication tables underlying Napier's rods, with a device to assist in adding up the partial products. There would be other attempts at this approach to direct multiplication over the next three centuries, running right into the twentieth century. However, as we will see, all proved rather clumsy, and when not clumsy to use, complex to make. Perhaps more important had been Schickard's insight that a series of interlinked gear wheels could be used to add and subtract, and furthermore, that a carry mechanism was possible.

It was this second focus which was to prove a more successful direction over the next several centuries. The time was ripe for thinking about the application of mechanisation to calculation. Its use to reduce the labour of addition was an attractive line of attack. Thus it was not surprising that only two decades after Schickard, a similar mechanical method of addition and subtraction (with some definite improvements) was rediscovered elsewhere - this time in France.

Blaise Pascal's Pascaline.

Blaise Pascal (1623–1662) had been only a 13 years old, when Schickard died but had already showed both a strong interest and flair for mathematics. His father Etienne, was a reasonably good mathematician in his own right, of noble birth and reasonably well to do, and when Blaise was 19 years old, with an appointment by Cardinal Richelieu to "Commissioner deputed by His Majesty for the Imposition and Collection of Taxes in Upper Normandy".[96] The pressure of work involved was enormous and Blaise was needed to help out in the extraordinary effort involved

96. Michael Williams, *History of Computing Technology*, p. 125.

in adding seemingly endless numbers as taxes were calculated, collected, paid and audited. As a consequence he turned what would prove to be his brilliant mathematical mind to the more practical task of constructing a mechanism to assist in addition. One of his heirs (chevalier Durant Pascal) would later claim that finding no artisan clever enough, Pascal himself trained himself with the skills to produce one of the machines with a few tools. However, this is very likely false. The clock-maker's guild was well established in Rouen, clock-makers had all the necessary tools, and further had an exclusive right to make any machine like a clock.[97] In any case, Pascal definitely did seek to wring the very best out the local artisans, and in this he is considered to have been very successful, producing machines which were not only useable, but works of the finest craftsmanship. A fully working replica of a Pascaline, crafted by Jan Meyer in Germany, and acquired for this collection in 2013, is shown in figure 3.16 below.

Figure 3.16. Working replica of a Pascaline,[98] style ~1650 (collection Calculant)

> *It was common knowledge that [Pascal] seemed able to animate copper, and to give to brass the power of thought. Little unthinking wheels, each rimmed with then ten digits, were so arranged by him that they could*

97. Jean-François Gauvin , *Habits of Knowledge: Artisans, Savants and Mechanical Devices in Seventeenth-Century French Natural Philosophy*, The Department of the History of Science, in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the subject of History of Science, Harvard University, Cambridge, Mass., USA, November 2008, pp. 229–45

> *give accounts [rendre raison] even to the most reasonable persons, and he could in a sense make dumb machines speak.* Jean Mesnard.[99]

As to the innovation in mechanism there is no evidence that Pascale had any knowledge of Schickard's invention. Nevertheless it was clear that he too was infected with the growing understanding that his invention could now be turned to reducing labour in new ways. His Pascaline turned out to be an adding machine, rather like that in the base of Schickard's clock, but with two important improvements (and one significant deficiency). As with the Schickard's machine, numbers were added by turning disks with a stylus which in turn turned interlinked gears. Pascale however experimented with ways to improve the practical functioning of this. Various details of the Pascaline mechanism, including diagrams in this collection from the famous Encyclopédie ou Dictionnaire raisonné of Diderot and d'Alembert in 1759 are shown in table 3.8, *Mechanism of the Pascaline*, below.

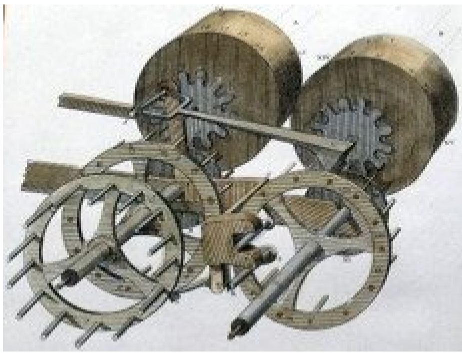
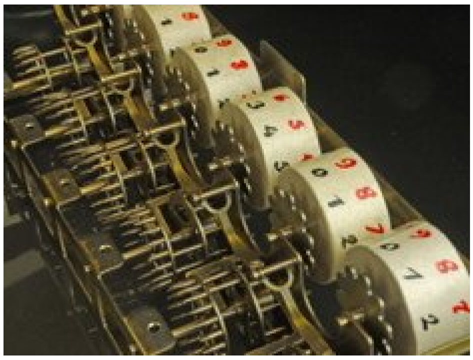
First, drawing firmly on the history of clock design, Pascale introduced gears which were minaturised versions of the "lantern gears" used in tower clocks and mills. These could withstand very great stresses and still operate smoothly. Second, he sought to achieve a carry mechanism which could operate even if many numbers needed to be carried at once (e.g. for 9999999+1 to give 10000000). He finally achieved this with a system where as each gear passed from 0 to 9 a fork-shaped weighted arm (the "sautoir") on a pivot was slowly rotated upward. As the gear passed from 9 to 0 this weighted arm was then released to drop thrusting an attached lever forward to rotate the gear ahead of it by one unit.

A deficiency was that the carry mechanism did not allow the process of addition to be reversed. Instead when subtracting a rather clumsy system had to be used of adding 'complementary numbers' (where a number -x is represented by (10-x), with 10 subsequently being subtracted). This was assisted by a window shade which can be switched between showing numbers or their complements.

Unlike Schickard's machine which is known only by documents, and which in reality would have had practical problems with the carry mechanism, Pascal sought to find profit from his invention. According to him he created some 50 of his calculators of varying design before settling on a final design. Some eight surviving Pascalines can be found in museums and private collections. He also documented his machine in a

99. Jean Mesnard (ed.), "Pascal, Entretien avec M. de Sacy sur Epictète et Montaigne", in *Pascal, Oeuvres complètes*, 4 vols., Desclée de Brouwer, Paris, 1964–1992, pp. 124–157 (translation from Jean Khalfa, "Pascal's theory of knowledge," in Nicholas Hammond (ed.), *The Cambridge companion to Pascal*, Cambridge University Press, Cambridge, 2003, p. 123.

Table 3.8. Mechanism of the Pascaline

	
Pascaline Mechanism diagram (1759) Diderot & d'Alembert[102] (collection Calculant)	Replica Pascaline mechanism with spoked lantern gears (collection Calculant)
	
Turret clock with lantern gears[103] from 1608	Replica fork-shaped carry mechanism (sautoir) (collection Calculant)

short pamphlet,[104][105] achieved what in modern terms would be called a patent for his invention - a right awarded by the monarch to exclusive production of the invention (thereby over-riding the clock-maker guild's claim to this) - and commissioned an agent (Prof. Gilles de Roberval) to sell it.[106]

Whilst Schickard's machine could multiply directly (using its ingenious incorporation of Napier's rods), Pascal's machine (despite his claim that it could be used for all operations of arithmetic) was primarily useful for addition, and with some mental effort, subtraction . What I argue elsewhere is a rather empty debate has at times erupted over whether Pascal or Schickard should be considered the more fitting candidate for 'inventor of the modern calculator'.[107] Nevertheless, it is Pascal's machine which has been referred to with reverance down the following centuries, not the least because a significant number were produced, it was mechanically ingenious, and some survive. A woodcut from 1901 of an accounting Pascaline, similar to that in the `Léon Parcé collection`, is shown in figure 3.17 below.

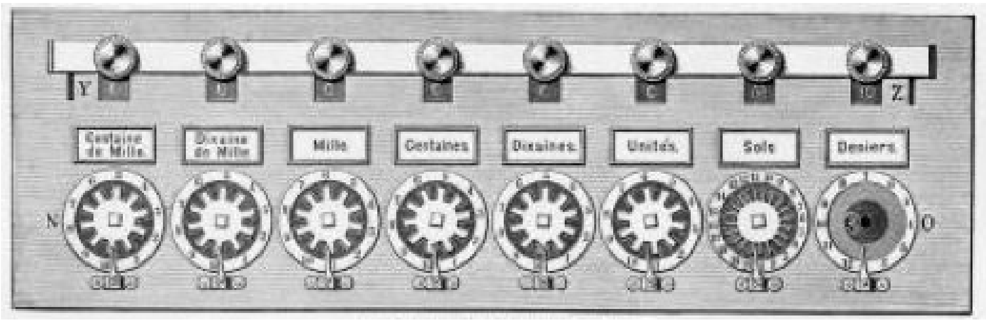


Figure 3.17. Wood engraved plate from 1901 depicting a Pascaline Calculator for accounting (~1642)
(collection Calculant)

Counting, Clocks, Colleagues and Courtly calculation

It is worth pausing to consider the reasons that Schickard and Pascal had launched into their geared calculator projects, almost certainly with no knowledge of each other's efforts, being the first known such projects in more than 1500 years. Notably,

---

104. `www.bibnum.education.fr/calculinformatique/calcul/la-pascaline-la-Â«%C2%A0machine-qui-rel>` `ève-du-défaut-de-la-mémoire%C2%A0Â»`
105. Blaise Pascale, "Lettre dédicatoire à Monseigneur le Chancelier sur le sujet de la Machine nouvellement inventée par le sieur B.P. pour faire toutes sortes d'opérations d'arithmétique par un mouvement réglé sans plume ni jetons, Avec un avis nécessaire à ceux qui auront curiosité de voir ladite Machine et s'en servir". Suivi du Privilège du Roy., 1645, `http://www.bibnum.education.fr/calculinformatique/calcul/la-pascaline-la-Â«%C2%A0machine-qui-relève-du-défaut-de-la-mémoire%C2%A0Â»` viewed 19 July 2012
106. Pascal, *ibid*, final page.
107. See `http://metastudies.net/pmwiki/pmwiki.php?n=Site.SchicardvsPascal`

whilst Schickard included a rendition of Napier's rods to aid multiplication, and Pascal worked to create a more effective carry mechanism, both in rather similar ways ultimately sought to mechanise addition and subtraction through a set of interconnected gears and dials. Of course there was no single reason why they embarked on such a similar task so close together in historical time. Rather a skein of factors were coming together to make such innovations appealing and therefore more likely to be addressed by people with the resources and ingenuity to do so.

(i) Perhaps most subtly, this was a time when philosophical inquiry, and the emerging practice of what would more commonly become known as scientific inquiry, were taking a more practical turn. There was a growing realisation that investigation which engaged with the natural world though exploration of how it behaved, could yield rich results. Notable in leading this idea was Francis Bacon, who in 1620 had written his *Novum Organum*, a strong argument that systematic empirical engagement of this type, could not but result in "an improvement in man's estate and an enlargement of his power over nature."[108] Implicit in this was a narrowing of the gap between science and technology, new ideas and application for betterment, and intellectual investigation, tools and technique. It was no less than a launch of "the idea of progress" which, as mentioned earlier, over subsequent centuries was to act as a reinforcing ideology for merchants and entrepreneurs, eventually helping sweep before them and the market much of the religious and customary authority of the aristocracy.[109]

(ii) As already noted, it was a time when clocks and clockwork were celebrated, with even the Universe being considered, at least metaphorically, as being a form of clockwork. And what clocks did was to count time. They used the rotational motion of geared wheels to count out seconds, minutes and hours, which were displayed on dials. The design required gears that could cycle (through 60 seconds or minutes) and during each cycle 'carry forward' a minute or hour. Whilst the approach adopted was a more incremental motion, the extension of such a mechanism to count units of 10s and carry did not require an impossible leap of insight. It is no coincidence, therefore, that Schickard named his device a 'calculating clock'.

(iii) Artisans, skilled particularly in the art of constructing clock mechanisms, existed with tools and workshops that could be turned to the task of constructing, similar, if differently configured and designed gear trains, dials and associated components. As already remarked, the importance of clocks is reflected in Pascal's choice of lantern gears for his Pascaline. Even his famous sautoir, whilst highly innovative and different in form, is reminiscent of the Verge escapement mechanism introduced into clocks from the late thirteenth century. In both a toothed mechanism was mechanically 'wound up' in a cycle and releasing at the correct moment in the cycle to control the motion of connected parts.

(iv) Each inventor had not only great intellectual ability but also a wide ranging

108. Francis Bacon, *The New Organum*, 1620, final paragraph, text reprinted at `http://www.constitution.org/bacon/nov_org.htm`, viewed 21 Jun 2013.
109. see for example, Bury, *The Idea of Progress*.

intellectual curiosity. Combined with this was personal motivation to seek to mechanise calculation. For Schickard it was an increasing interest in discovery and application of new knowledge, found in a dispersed, small, but communicating network of people interested in all manners of philosophy and theology. It included natural philosophers such as Kepler, who had an increasing need to utilise and overcome the drudgery of large numbers of calculation. Napier through his rods and logarithms, had provided means to greatly assist multiplication. But reducing the drudgery of associated additions and subtractions was emerging as something that would be valued. Pascal's initial motivation was to assist his father in his extensive revenue collecting duties. But Pascal was also on a rapid rise as a natural philosopher and thinker in his own right, where the devising of a ground breaking mathematical instrument also stood to be valued by the network of other thinkers in which he and Schickard were participating.

(v) The network in which Schickard and Pascal engaged was could not be composed, in any case, of any people. They had to be well educated and with time to follow these pursuits. And that required that, almost without exception, they would be well connected to, or members of, the highest ranks in society, that is the nobility. From this point of view, the products of their work were likely to be intended to find favour with others of that rank.

Consistent with this, many of Pascal's machines would end up, not in the hands of practitioners of mathematically intense duties, but as curiosities on the shelves and in the cabinets of persons of eminence. The names indicating the provenance of some of the *surviving Pascalines*[110] - Queen of Sweden, Chancelier Séguier, Queen of Poland, Chevalier Durant-Pascal - are consistent with this. But perhaps equally so is the beautiful workmanship and decorative working of materials which are characteristic of these instruments.

In a fascinating thesis,[111] and subsequent published book chapter,[112] Jean-François Gauvin develops a multi-stranded analysis of the role of scientific instruments, including the Pascaline, and their creators and use in the seventeenth century. Key to this are conflicts and resonances between continuities in cultural habit, and social, philosophical and ideological challenges to them that were beginning to gain force at the time.

Challenges such as Bacon's *Novum Organum* reflected the beginnings of a discomfort with knowledge founded on the verities of ancient Greek knowledge, church orthodoxy and aristocratic tradition. Of course both Schickard and Pascal, sought and enjoyed the indulgence and support of aristocratic and religious sponsors and Pascal in particular would use his machines as much as a way of attracting that as for any commercial gain. But the transformation reflected in Bacon's call was to progressively

110. metastudies.net/pmwiki/pmwiki.php?n=Site.SurvivingPascalines
111. Gauvin , *Habits of Knowledge*
112. Jean-François Gauvin, "Instruments of Knowledge," in the *Oxford Handbook of 17th-Century Philosophy*, ed. by Desmond Clarke and Catherine Wilson, Oxford University Press, Oxford, UK, 2011, pp, 315–337.

reposition the man of substance (a "gentleman", or in France "l'honnête homme")[113] in relation to some tools such as these. Building on the existing prestige of clockwork and the growing understanding that intellectual advance was tied to mechanical engagement with the natural world, these devices could be seen to potentially restore the separation of the intellectual world of l'honnête homme from the drudgery of the 'mechanical' act of routine calculation.

Having said that, the machines were expensive (about of a third of a year's average wages of the time).[114] Pascale in his surviving *Advice* on the use of the Pascaline promised that the machine could "perform without any effort whatsoever all the arithmetical operations that had so often worn out one's mind by means of the plume and the jetons".[115] But in truth the use of it required a good knowledge of arithmetic to be used effectively (especially for division and multiplication, but even for subtraction). At best it was a mechanical substitute for what otherwise would have to be written down, save that it could perform addition with a little practice, and simple subtraction (where the result was not negative) with a little more. As Balthazaar Gerbier described the Pascaline in a letter to Samuel Hartlib in 1648:

> a Rare Invention farre saught, and deare baught: putt them in the Storre house was the old Prince of Orange wont to saye and lett us proceede on the ordinary readdy [ready reckoning] way.[116]

But like rare books need not be read to add luster to the shelf, the machines needed not necessarily be used to add something to the owner. Basic mathematical skills were far from universally held, even amongst the nobility, and certainly amongst gentlemen. From this point of view, even if one could not add and subtract reliably in one's head, let alone multiply using jeton (calculi) and pen, the potential allure of such a calculating machine was that by owning it, one could come to at least be seen to have an intimacy with such literacy. In these senses, as Gauvin argues, "Even though Pascal invented the machine to alleviate his father's headaches as a royal tax collector, financiers and merchants, who tallied large amount of numbers, were not especially in Pascal's mind. The pascaline was more than a mechanical contraption useful for business: used properly, it could bestow l'honnête."[117]

113. A particularly seventeenth century development, positioning those of standing beyond those in the nobility, as still somehow aligned to that status by demonstrating an ideal style of human qualities where the person combined a measured quality of heart and mind with, amongst other things, a broad grasp of current intellectual concerns, integrity, and the cultured politeness of courtiers.
114. From *Les collections du musée Henri Lecoq*, volume V, "Les Machines Arithmétiques De Blaise Pascal", the cost of a machine was about 100 livres at a time that the average wage was about one livre per day. `http://calmeca.free.fr/calculmecanique_php/rubriques/Fichiers_Blaise_Pascal/Fichiers_historique/Pascaline_histoire.php?lang=eng`, viewed 25 June 2013]]
115. Pascal, introduction to "Avis nécessaire" in *Lettre dédicatoire à Monseigneur le Chancelier*, p. 9.
116. Balthazaar Gerbier to Samuel Hartlib, 4 October 1648, in S. Hartlib, *The Hartlib Papers: A Complete Text and Image Database of the Papers of Samuel Hartlib (c.1600–1662)*, ed. J. Crawford, Ann Arbor, Michigan, 1995, ephemerides (1655) part 1, cited in cited in Ratcliff, "Samuel Morland and his calculating machines", p. 174.
117. Gauvin, *Habits of Knowledge*, p. 117.

Even so, despite Pascal's best efforts, in particular stressing the similar complexity of the mechanism of his machine and precision required of its workmanship to that esteemed in clocks and watches, the Pascaline found no broad market. As Gauvin, puts it "Unlike watches, the pascaline was much heavier and thus not easily portable; unlike table-top clocks, it was not as ornate and could not do anything on its own. The pascaline was a luxury item that fit no preestablished fashionable categories and could not initiate by itself a new one. It became a rarity, and like most rarities it found its place in cabinets of curiosities."[118]

The above provides some basis for understanding what followed: a series of developments and experiments in mechanical calculation, few of them seen abstractly providing much real advantage over traditional pen and jeton for doing arithmetic, but each embodied in beautifully worked prototypes, often frequently being found on the shelves or in the cabinets of curiosities of the nobility and others of standing, whether in Germany, France or England. Since details of these are available elsewhere[119] we will rely on objects documented in this collection to simply act as signposts. In particular, two inventors following Pascal, Leibniz and Moreland, will be briefly considered, each of which illustrates substantially the above contention.

The inventions of Morland and Leibniz.

The multiple potential attractions of such mechanical embodiments of arithmetic can be seen to be at work over the next several centuries. From Schickard and Pascal other inventors sought in one way or another to make progress over the known work, at least of Pascal. One of these was Leibniz in Germany, and the other Morland who created the first English calculator. Each made a further contribution to the art and whilst the practicality of their inventions, even at the time, remains in contention, each gained satisfaction from their efforts for one or more of the diverse reasons mentioned above.

Samuel Morland (1625–95) - son of an English clergyman - had a complex life in a difficult time. At the age of 24 (the year he matriculated from Cambridge) he experienced the English revolution with the execution of King Charles I. Then he began work for Cromwell as a courtier-inventor a year later primarily providing intelligence through methods of postal espionage (intercepting, opening, decrypting and interpreting, and re-sealing mail). In the course of this, he was almost killed by Cromwell on suspicion of overhearing a plot to lure to England and kill the exiled Charles II,[121] son of the executed King Charles I. Indeed Morland had overheard the plot and subsequently reported it to Charles II's supporters. After Cromwell's death (in 1658) Morland was able to manage the delicate transition to service under the newly restored King Charles II and was knighted by him in 1660 and made a Baronet soon after.[122]

118. ibid, p. 230.
119. see for example, *Rechenmaschinen-illustrated*[120]
121. James Orchard Halliwell-Phillipps, *A brief account of the life, writings, and inventions of Sir Samuel Morland: master of mechanics to Charles the Second*, E. Johnson, Cambridge, 1838, p. 7.
122. ibid, p. 8.

In the course of these events Morland, whilst certainly comfortably provided for (not the least after he married a Baroness), complained of the failure of his positions to yield wealth saying: 'Now finding myself disappoynted of all preferment and of any real estate, I betook myself too the Mathematicks, and Experiments such as I found pleased the King's Fancy.'[123] On the basis of his mastery of engineering (in particular, fluid systems) Morland was eventually granted a newly created position of "Master of Mechanicks" to the King, and later was made a gentleman of his Majesty's privy chamber.[124] His inventions, thus played a two-fold role, first in his search to establish a role for himself in the context of the new Court, and second to supplement his wealth by seeking commercial success. In particular, he hoped for a lucrative outcome to devising mathematical instruments and selling them.

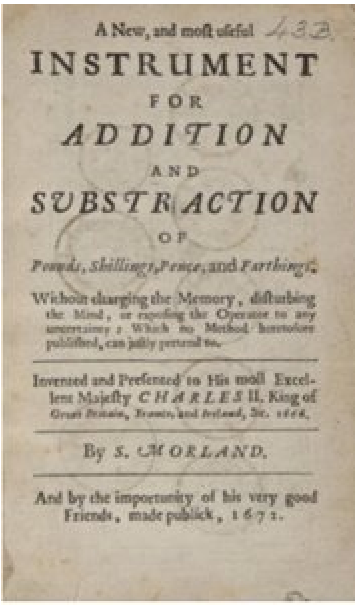
Morland had already seen a Pascaline, probably when on a diplomatic mission to Queen Christina of Sweden, a supporter of the sciences who in 1649 had been presented with a Pascaline (similar in looks and identical in function to the replica in this collection).[125] He had also taken part in a diplomatic mission during which he stopped off at the court of Louis XIV for over a month, so he may well have learned more of the Pascaline, and made contact with scholars and associated mechanics at that time.[126] He was thus aware of the potential allure of such inventions. Over the next few years he devised three different calculator designs: an adding machine (see figure 3.18 below), a multiplying device (see also figure 3.19 below), and an instrument with moveable arms for determining trigonometric relations. Several examples of Morland's calculating devices are still in existence, in particular in museums in London and Italy. Not only did Morland design the machines but he wrote a book on the use of the instruments. Published in 1672 this can be considered the first known English language (and arguably the earliest surviving) 'computer manual'. (A copy of this rare book, shown below in table 3.9, *Morland's Illustrations*, is in this collection).

In essence the adding machine was a simplified adaptation of the Pascaline's dials (turned by stylus), but without any carry mechanism (so that carrying had to be done by hand). Adding (or subtraction by an opposite rotation) was input through the larger wheels. Each larger wheel engaged with the small wheel above to register a rotation and thus accumulated a 1 to be carried. As with the Pascaline different input wheels were provided for different units (whether units, tens, hundreds, etc, or pounds, shillings and pence).

The multiplying machine was simply a mechanised representation of Napier's rods. In this sense it followed in the footsteps of Schickard although it is doubtful that Morland

123. Autobiographical Letter to Dr. Thomas Tenison, 1689, reprinted in H.W.Dickinson, *Sir Samuel Morland: Diplomat and Inventor 1625–1695*, Cambridge,1970.
124. J. R. Ratcliff, "Samuel Morland and his calculating machines c.1666: the early career of a courtier–inventor in Restoration London", *British Journal for the History of Science*, Vol 40, number 2, June 2007, pp. 159–179; and Halliwell-Phillipps, *A brief account of the life, writings, and inventions of Sir Samuel Morland* p. 9.
125. Diana H. Hook and Jeremy M. Norman, *Origins of Cyberspace* Novato, California, 2002, p. 111
126. Michael R. Williams, *A History of Computing Technology*, p. 137.

Table 3.9. Morland's Illustrations

 A New, and most useful 43B INSTRUMENT FOR ADDITION AND SUBSTRACTION OF Pounds, Shillings, Pence, and Farthings; Without charging the Memory, disturbing the Mind, or exposing the Operator to any uncertainty; Which no Method heretofore published, can justly pretend to. Invented and Presented to His most Excellent Majesty CHARLES II. King of Great Britain, France, and Ireland, &c. 1666. By S. MORLAND. And by the importunity of his very good Friends, made publick, 1672.	
Morland 1672 cover page "Instrument for Addition and Subtraction . . . "	Morland 1672 multiplying instrument (collection Calculant)

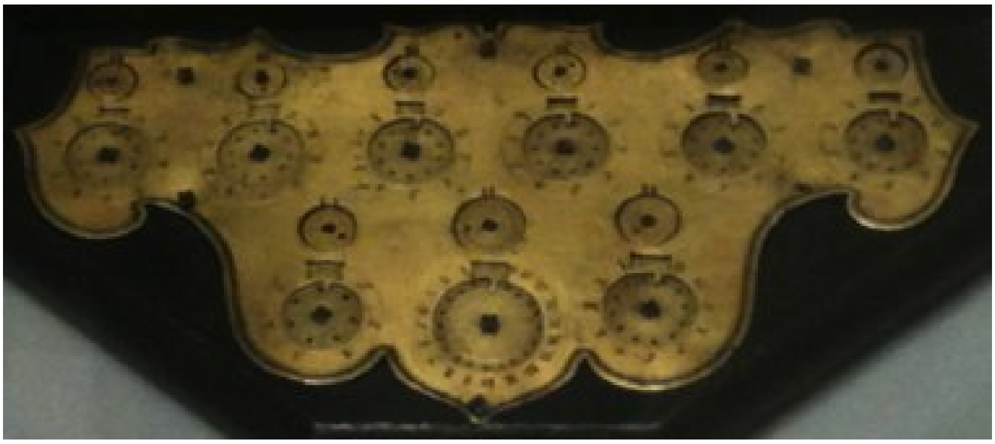


Figure 3.18. Morland Adding Machineadapted to the then Italian currency (Istituto e Museo di Storia della Scienza, Florence - Photos by Calculant)

Figure 3.19. Morland Multiplying Instrument(Istituto e Museo di Storia della Scienza, Florence - Photos by Calculant)

would have known of Schickard's work. In Morland's machine the ten Napier rods were replaced by ten rotatable disks, with the corresponding Napier numbers inscribed on their circumferences (with units and tens of the rods placed diametrically opposite each other). To multiply the operator took the disks corresponding to the number to be multiplied, and lifted the lower windows plate, to placed the disks on posts. A key was then turned until a sliding indicator matched the multiplier (being a number from 1 to 9). Each turn of the key rotated the discs and advanced them under the windows producing a display of the partial products of the multiplier. The partial products then had to be added which Morland suggested could be done with the aid of his adding machine.[127]

These machines were variously received as "those incomparable Instruments"(Sir Jonas Moore),[128] "not very useful" (Henri Justel),[129] or "very silly" (Robert Hook).[130] But in terms of obtaining patronage on the one hand (not only in England but also from the Medici in Italy), and at least some sales to those men and women with wealth but not much knowledge of addition or the multiplication tables, the instruments served at least some of the needs of their inventor. That being so, they perhaps provided more reward to both maker and purchaser in terms of status than they returned financial benefit for the former, or enhanced arithmetic capability for the latter.

Gottfried Wilhelm Leibniz (1646–1716) had a life filled with so many ambitions, such high profile conflict, and so much capacity that his biography can seem overwhelming. He has been described as ". . . an indefatigable worker, a universal letter writer (he had more than 600 correspondents), a patriot and cosmopolitan, a great scientist, and one of the most powerful spirits of Western civilisation."[131] At the same time his irascible

127. Ratcliff, "Samuel Morland and his calculating machines", p. 168.
128. J. Moore, *A Mathematical Compendium*, London, 1681, p. 21.
129. Justel to Oldenburg, 27 June 1668 and 15 July 1668, in Hall and Hall, cited in Ratcliff, "Samuel Morland and his calculating machines", p. 175.
130. Robert Hooke, diary, 31 January 1672/3, cited in Dickinson, *Sir Samuel Morland*
131. Encyclopaedia Britannica, `http://www.britannica.com/EBchecked/topic/335266/Gottfried-Wilhelm-Leibniz/4131/The-Hanoverian-period`, viewed 25 June 2013.

personality was such that by the time he died, his contemporaries were said to feel a sense of relief and only his secretary attended his funeral.[132] His father was a professor of moral philosophy who died when Leibniz was only six years old, but he grew up in the surroundings of his father's imposing library from which he derived much satisfaction. He gained his Doctorate of Laws (on *De Casibus Perplexis*, i.e. *On Perplexing Cases*) at the age of 20 from the University of Altdorf in Nürnberg.

The scope of Leibniz's subsequent work was extraordinary encompassing poetry and literature, law, political diplomacy, work to unify all knowledge in part by bringing the scientific societies together, a similar desire to bridge the gap between the Lutheran and Catholic church in particular, and all churches in general, and more enduringly, his powerful initiatives in science and mathematics. The products of his work in mathematics included the development of binary arithmetic, methods of solving systems of linear equations, and either the discovery of calculus (priority in this was contested by Newton) or at least the modern notation used for it. Central to his work was a belief that if a logically defined ('mechanical') algebra of thought could be developed then truths could be automatically generated and proved. (This quest can be found stretching back to the Stoics, and forward to the later work of Gottlob Frege, George Boole, Bertrand Russell, Kurt Gödel and many others). Given this enticing objective it is not surprising that, having heard of Pascal's calculating machine when he visited Paris in 1672 on a diplomatic mission (later to be elected to the French Academy of Sciences), Leibniz decided to turn his hand also to creating a calculating machine.[133]

Almost certainly Leibniz did not have a chance to use a Pascaline or he would have discovered that an early idea that he had, to automate multiplication by placing a mechanism on top of the Pascaline to simultaneously move its input "star wheels", would conflict with the machine's internal mechanism. His second attempt was much more original. Although unlike Pascal he was never able to properly automate the carry system, he developed a machine which could more faithfully replicate the pen and paper methods not only of addition, but subtraction, multiplication, and with some ingenuity, division. The first and most enduring innovation was a new way to input numbers by setting an accumulating cog to engage with a "stepped drum".

The drum had 9 radial splines of incrementally increasing length and could be turned with a crank handle. An accumulating cog could be slid along an axle parallel to the drum and when the drum turned through a full rotation depending on where the cog engaged with the splines of the drum,the cog would be turned by anything from 0 to 9 of the drum's splines thus accumulating 0 to 9 units of rotation. An example from a C20 calculator is shown in figure 3.20 below.

Several such accumulating cogs with their drums were put side by side, corresponding

132. Williams, *History of Computing*, p. 136.
133. *Gottfried Wilhelm von Leibniz*, School of Mathematics and Statistics University of St Andrews, Scotland, October 1998, `http://www-history.mcs.st-andrews.ac.uk/Biographies/Leibniz.html`, viewed 25 June 2013.

Figure 3.20. Twentieth Century step drum following the same principle as Leibniz's conception[134]

to units, tens, hundreds, etc, and once a number was set (e.g. 239) it could be multiplied (e.g. by 4) by turning the crank through a full rotation the corresponding number of times (thus adding 239 to itself 4 times to give 956). The capacity to add a multi digit number to itself repeatedly gave the machine the capacity to multiply by simply turning a handle, which was a considerable advance over the Pascaline. Further, the crank could be rotated in the opposite direction to produce a subtraction.

As with Moreland's multiplication machine it was now possible to introduce a carriage which would be mechanically advanced (by means of a second crank handle at the end of a screw thread) to allow multiplication by more than single digit numbers to take place. The Pascaline had had no such mechanism and so the equivalent had had to be done by recording intermediate results with pen and paper. The deficiency in Leibniz's machine, however, as already indicated, was that unlike the Pascaline, Leibniz had been unable to devise a robust carry mechanism able to handle either multiple carries across several output dials at once. As a result, the machine was far from perfected. It seems that only one was made in 1674 (by M. Olivier, a French clockmaker, working under instruction from Leibniz), that a few years later it was sent Herr Kastner in Göttingen (Lower Saxony) for repairs, and was later stored in an attic at Göttingen University where it was not recovered for 200 years. A diagram of the Leibniz calculator from 1901 is shown in figure 3.21 below, and the actual surviving machine is shown in figure 3.22 also below.

The road forward

Multiplication tables and various tabluated functions (notably logarithms and trigonometric tables), and various physical embodiments of those notably in various forms of Napier's rods, sectors and scales, would increase in use as need and access to education in their use broadened. But could a machine be constructed that would make the mathematical literacy required in the use of these, and traditional methods of arithmetic using pen, paper, and calculi obsolete? Clearly the answer was "not yet".

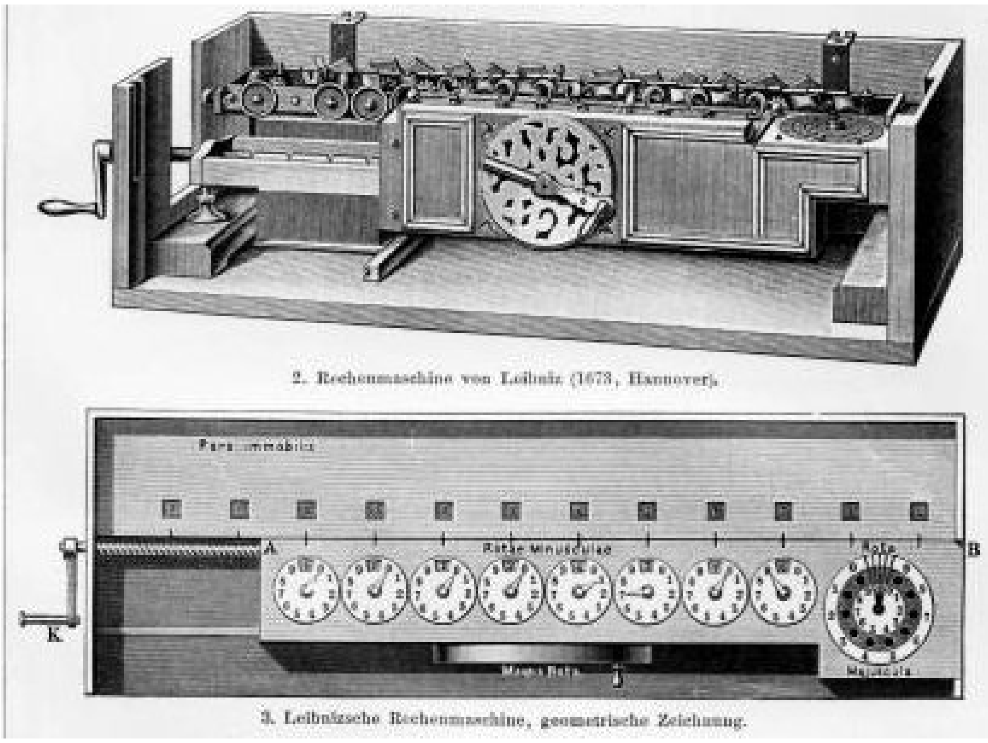


Figure 3.21. Wood engraved plate from 1901 depicting the Leibniz Calculator (1673) (collection Calculant)

Leibnitz's machine formed the final piece of a tryptage (with Pascal and Schickard) of foundation pioneering seventeenth century mechanical calculators. Together these constituted foundation stones in the subsequent development of a wide range of other calculational technologies over the next two centuries. Progressively these overcame many of the more apparent limitations in these foundation devices, yet they shared also one other characteristic. Despite hopes that may have been held by their inventors, none of them proved to be more than prototypes in the sense that their destiny would be as curiosities of great interest, perhaps prized by the few who might get hold of one, but for a variety of reasons (cost, capabilities, ease of use) of little broader practical importance. The reasons we have canvassed for constructing them, and possessing them, would remain pertinent, but the other possibility, that they might be cheap enough and be sufficiently intuitive in use to create a notable increase in mathematical facility in a widening group of users, would remain a dream, not to be realised, until Thomas de Colmar, two centuries after Leibniz, also utilised the step drum to open the door to the commercial production and widening public use of mechanical calculators.

Figure 3.22. Surviving Leibniz Calculator recovered from the attic of Göttingen University. Reproduced with permission from the Gottfried Wilhelm Leibniz Bibliothek.

Chapter 4

The Late Modern period

4.1 Part 3. The Advent of Commercial Mechanical Calculation.

4.1.1 A time of change.

The late modern period, spanning the nineteenth century through the two world wars of the twentieth century and ending roughly in the middle of that century, was a time of enormous economic, technological, cultural and political change. The role of calculators, from one point of view, was a comparatively minor part of this time of panoramic and turbulent change, and yet, it also was profoundly shaped by it, and helped facilitate its development.

As already suggested, none of these devices that were created during this period sprung from their inventors' minds completely unprecedented. The roots spread back into the early Modern period from the earliest forms of embodied calculation (for example, the use of calculi). The period following the inventions already mentioned of Schickard, Pascal, Moreland and Leibniz were followed by a multitude of devices built on similar basic principles, but all with the limitation that they were not widely taken up because as yet need had not developed to resonate with the limited capabilities and often high cost of the inventions. Nevertheless, as time passed a web of developments would continue to emerge that would eventually create that moment when such a resonance might take place. Central to this was the use to which these devices might be put.

Usefulness arises in a context, and the social, political, economic and technological context was in a process of unprecedented transformation across Europe. The late C18 was characterised most dramatically with the execution of Louis XVI, in 1793, by a transition between the power of the ancienne regime to that of new political forces. Into the following century across Europe the emerging power of merchant and market increasingly swept aside the old aristocracy in the shaping of the politics, economics and infrastructure of what, in a process evolving from the Treaty of Westphalia (1648) had emerged as 'sovereign' nation states. Improving systems of transportation facilitated a process of ever intensifying trade across and between states was involving monarchs, peasants, artisans, merchants and financiers alike.[1] Ever more efficient processes of printing and dissemination facilitated production and dissemination of ideas. A period of comparative peace between nations in the late C18 was marked by a revolution in agricultural production. But this was just one way in which a synergy between scientific and technological innovation was finding purchase within transforming systems of production, commerce, governance and more broadly lived experience. In addition, science was beginning to be harnessed to actual production. Steam replaced horse, and then from the mid C19 to the mid-C20, electrical networks spread across Europe and then much of the rest of the modern world allowing the introduction of many new technologies.

1. For more on this see Camilleri and Falk, *Worlds in Transition*, pp. 134–44.

Accompanying this the understanding of the world, and even the conception of meaning in existence, was being transformed in conjunction with the emergence of new scientific insights in mathematics, physics and astronomy. As Werner Heisenberg, a founding physicist in quantum mechanics has summarised it:

> In the period that followed, science began its victorious march on a broad front even into those distant realms of nature which could be entered only by technology .... Even the phrase 'description of nature' lost more and more of its original significance of a living and meaningful account of nature. Increasingly it came to mean the mathematical description of nature.[2]

Not only was the conception of nature being transformed through a more technical, and indeed mathematical account, but new literacies and cognitive skills were now required. Skills were now more broadly required to deal with an economy that was ever more shaped by the market, production that was ever more shaped by science, and products whose use required continual processes of cultural learning. Mass education in reading and basic mathematical skills were now an increasing necessity. Steadily the period of "childhood" was being extended (including, eventually, the invention of the "teenager") to allow an extended period of socially conceded time in which this personal development could take place.

Finally, technological development and the role of science was greatly accelerated by the two world wars, which became wars of science, as evidenced by leaps in aviation, rocketry, electronic communication, encryption, and detection - to name just a few - and cumulating in the harnessing of the process of nuclear fission itself to bomb Hiroshima and Nagasaki. Pressed forward by the onward thrust of market and military priority, the systematic research, development and deployment of new techniques, powerful precision tools, and diverse materials, often emerging in new goods, produced through evolving processes of mass production and social organisation, became a constant theme of the times.

It was then in this remarkable period of dynamic change that the insights into calculation technology finally found purchase.

4.1.2 Simple devices

As the above suggests, historical periods do not easily fit together as simply defined blocks of time with nice clear boundaries. Rather they are useful labels for different times of change, which whilst usefully distinguished, overlap each other to allow for the transitions which take place across them. The process of technological change in particular, in any particular period, is layered over, and has roots reaching back to the innovations which precede it. Indeed, not only did prior inventions provide a base on which new more commercially successful devices might be built, but also old

2. Werner Heisenberg, *The Physicist's Conception of Nature*, Hutchinson Scientific and Technical, London, 1958, p. 11.

methods could be persistently found in use sometimes right through the late Modern period. One of the most common of these, which may still be found in use in some places, was ready reckoners.

Ready reckoners

Ready reckoners were in widespread use by shopkeepers and many others from the C17 on. They presented tables of precalculated results of many kinds of multiplication and division, addition of fractions, important constants, unit conversions, logarithms, and much else. Frequently they included the much needed calculations of the price for multiples of an item for sale, or per unit of weight and could also be used to look up the calculations needed for wages and interest and .[3] One such, from 1892 is “Ropp’s Commercial Calculator: a practical arithmetic for practical purposes, containing a complete system of useful, accurate, and convenient tables, together with simple, short, and practical methods for rapid calculation”, published in the US and used widely into the early C20. This one, shown below in figure 4.1, is the 1892 World’s Fair Edition containing 128 pages of useful mathematical facts, formulas and tables.

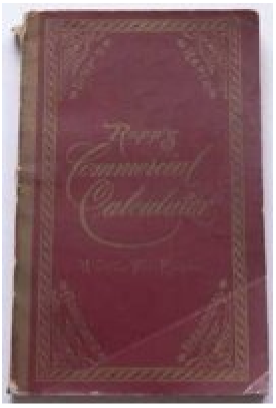


Figure 4.1. 1892: Ropp’s Commercial Calculator
(collection Calculant)

The dominant role of ready reckoners in this period is indicated by M. Norton Wise’s comment in relation to Victorian England that:

> By the 1860’s, the favorite device for lessening the work of the computer (human), the mathematical table, had become an object whose dizzying rows of printed figures would fascinate the Victorian public. These tables displayed the limitless fecundity of numbers, and transformed them into a commodity that would bring the power of calculation within the reach of the ordinary citizen. The centrality of tables of numbers and calculation to mid-Victorian life was famously portrayed by Charles

3. see Bruce O.B. Williams and Roger G. Johnson, “Ready Reckoners”, *IEEE Annals of the History of Computing*, October-December, 2005, pp. 64–80.

Dickens' character Thomas Gradgrind, who always had a rule and a pair of scales and the multiplication table in his pocket.[4]

Lookup tables in new form

Other early new mathematical devices to gain broader use were simple adaptations of the sorts of tables that might be found in ready reckoners. There was an ever more extensive need to be able to measure and compare especially in the expanding processes of trade and commerce. The bane of earlier centuries (and even still) is lack of standardisation across localities in which trade took place. In particular, quantities were often not measured in the same units. As a result, special purpose devices, basically just forms of mechanical look-up tables were devised to enable conversion, needed not the least since performing associated conversion calculations each time was not only time consuming but beyond the skill of many who might need them. A particularly stark need for such devices was created, following the French Revolution, when the Revolutionary Government introduced metric measures. A "Convertisseur" from (1780–1810) designed by clock maker Gabrielle Chaix in Paris to assist in converting between the old measure of distance (the "Aune") to the new metric metre that was introduced in 1791, is shown in figure 4.2 below.

Figure 4.2. ~1790: Conversion device (collection Calculant)

The resemblance between the design of the above and that of a prestigious gentleman's pocket watch is probably not coincidental. Mechanisation can come in the most basic forms and still carry the allure of the new whilst usefully substituting for otherwise necessary mental skill. Many conversion devices of similarly simple construction continued to be manufactured into the second half of the twentieth century.

4. M.N. Wise, *The Values of Precision*, Princeton University Press, USA, 1995, p. 318.

Available principles

It is useful to recapitulate the available principles which had been discovered upon which devices for more sophisticated arithmetic tasks could be devised. Key to all of these was the idea, already embodied in calculi and counting tables, or the abacus, that addition of numbers represents the addition of units of something physical. Bearing this in mind, adding units of motion in space could do the same job, whether the movement was units of:

- movement of a point along a line (implemented for example by a sliding rod, moving chain or strap, or something similar)
- rotations of wheels and cogs
- something else that can be moved (for example, although it was barely used,[5] the height of columns of water).

Subtraction could be achieved by units of motion in the opposite direction, or where that was not possible, the addition of complementary numbers.

Multiplication could be achieved by repeated additions, and divisions by repeated subtractions. Alternatively tables of multiplication (or logarithms) could be utilised to replace these steps.

Examples utilising these analogue motions to aid arithmetic were already available from as early as the C17.

- The slide rule had used the motion of sliding along a straight line to add distances representing a logarithmic scale.
- The devices of Schickard, Pascal and Leibniz had used rotation of dials to achieve addition of rotations. Schickard's machine had the capacity to subtract by turning the dials backwards, but had not fully resolved how to carry from one dial to another.
- Pascal had developed a very efficient carry mechanism but at the expense of the dials not being able to be rotated backwards. Both of these relied on a stylus for input.
- Leibniz had developed his step drum to improve the input process and repeated additions were facilitated by the capacity of the machine to be turned with a crank handle. But his carry mechanism needed further development to work when this occurred.

It would be tempting to present the subsequent development devices as a single linear track of improvement from these promising beginnings more than a century before. But this would be artificial. Rather a genealogical tree of innovations spread out from these earlier inventions as a niche for their use began to be uncovered.

5. An exception is the ingenious MONIAC hydraulic computer used to demonstrate macroeconomic theory. See for example, Anna Corkhill 'A superb explanatory device' *University of Melbourne Collections*, issue 10, June 2012

Simple linear devices

One such line of development was based on the possibility of adding linear displacements of strips of metal. This type of device, first developed in the 17th Century by inventors such as Claude Perrot (1613–88) was popularised by J. Louis Troncet, a French inventor who created his Arithmographe, later to be known as the Addiator (of which there is one in this collection). A bewildering number of different designs around this idea appeared in the late C19 and early C20. Some used strips or rods, and some used chains which were more versatile. A rather beautiful Locke Adder which uses strips moved by knobs, from 1905–10, sold by Clarence Locke, is shown in figure 4.3 below.

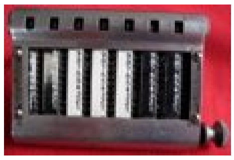
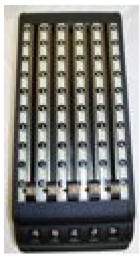
Figure 4.3. The Locke Adder
(collection Calculant)

Other simple linear devices in this collection are shown in table 4.1, *Simple Linear Calculators*, below.

Table 4.1. Simple Linear Calculators

Date	Description	Maker	Type	Device
1910–20s	Comptator (9 col)	Schubert & Salzer	Chain	

Continues…

~1915	Golden Gem Automatic Adding Machine	H. & H. Goldmann	Chain	
1920	Addiator Troncet adder	Ruthard & Co	Chain	
1920	Addo adder	Aktiebolaget Addo	Rod	
1922	Scribola 10 column printing Adding Machine	Addiator Gesellschaft	Rod	
1937	Add-O-Matic Adding Machine	Allied MFG. Co.	Chain	

(collection Calculant - all above)

The above relied either on the movement by a stylus, or knobs, of strips, rods or chains to achieve an addition. Some (such as the Locke adder) used a simplified form of complementary arithmetic for subtraction. Others, such as the Golden Gem, allowed reverse motion for direct subtraction. Others, such as the Addiator, could be turned over to perform the subtraction on the reverse side. Most (but not the Locke Adder) had a simple provision for carries. And some (such as the Scribola (which also had a very early approach to printing its result on a paper strip) had clearing levers to bring its display back to zero for the next calculation. All however, whilst affordable, were cumbersome and slow to use since each digit had to be individually entered by performing a sliding motion. It was much quicker to add the numbers on paper using mental arithmetic. But, of course, where that capability was lacking, or where it was insufficiently well practised and the effort of doing it was consequently considered tedious, then these devices began to find a market in the first half of the C20. Thousands were produced and sold across the more industrialised countries of Europe, the British Empire and the USA.

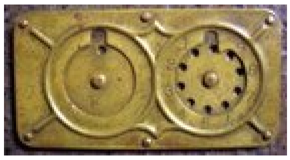
Simple rotational devices

Similar simple devices were developed which utilised simple rotation of cogs, with a crude carry process (usually along the lines of that used by Schickard). They relied for their success on having but a small number of interacting cogs, so that the carry mechanism would not jam, or adopting the use of springs (in place of Pascal's weights) which would store rotary motion to then be utilised when a carry was required. The device shown below in figure 4.4 was the first model made by C.H. Webb (from New York) who began marketing it in 1869. It is made of brass and is set on a heavy wooden base.

Figure 4.4. 1869:Webb Patent Adder and Talley Board
(collection Calculant)

Other simple rotational adding devices in this collection are shown in table 4.2, *Simple Rotational Calculators*, below.

Table 4.2. Simple Rotational Calculators

Date	Description	Maker	Type	Device
1890–1900	A. M. Stevenson Adder	Joliet, Ill.	2 Wheel	
1910	Adall Adding Machine	Dreyfus and Levy	Concentric toothed disk and groove	
1922	Conto Mod C Adding Machine	Carl Landolt	Rotating Knob	
~1946	Lightning Portable Adding Machine	Lightning Adding Machine Co. L.A.	7 Wheels	
1968	SEE Demonstration Adding Machine	Selective Educational Equipment Corporation	4 Wheels	

Continues...

(collection Calculant - all above)

The main advantage of these devices was that they were cheap to manufacture and could be easily mass-produced. A lot of innovation is evident in the detail of these machines. A particularly simple approach to the issue of carrying was addressed in the Addall from 1910, a circular proportional adder which used rotations of a wheel to wind on successive numbers whilst a ball bearing, moving in a spiral groove, tracked the successive rotations.

Early experiments with keyboards can be seen below in table 4.3, *Early keyboards*, below. Each of them is very different in appearance yet all three - the Spalding Adding machine of 1884 (of which only a very few were produced, and fewer still survive), the Centigraph of 1891 (also extremely rare), and the more conventional looking layout of the Addix Adding Machine of 1905 - all had a similar principle of operation. In each case the depression of the different number keys actuates a lever to move a gear wheel a corresponding number of notches, thus adding the number. Each of them of course achieves this in their own way. And they have very different appearances with quite different forms of visual output of the accumulated number - dials (for the Spalding), a two digit display and arrow pointing out the 100s (up to 500) for the Centigraph, and a more conventional set of windows for the Addix indicating that there are many ways that a simple basic principle can be expressed. None of them, however, was particularly useful given their clumsiness of operation (for example, in clearing a result), and limitations in function (essentially to addition).

Figure 4.5. 1884: Spalding Adding Machine
(collection Calculant)

The most obvious other development beyond the subtleties of carry mechanism was the developments in the use of materials. Here we see the heavy two wheel mechanism of the Webb Adder, in brass and wood, counterposed against the Adix Adding Machine which for the first time incorporates aluminium amongst its 122 parts, whilst the SEE Adding Machine which is entirely composed of plastic, has a Pascaline-like weighted carry mechanism, but is built so lightly utilising a

Table 4.3. Early keyboards

Date	Description	Maker	Type	Device
1891	Centigraph Adding Machine	Am. Add. Co.	Key input, lever & gear	
1903	Adix Adding Machine	Manheim	Key input, lever & gears	
(collection Calculant - all above)				

materials evolution over three centuries, that a much simplified approach was by now implementable.

A particularly elegant, if made of brass and thus heavy, device was the Swiss Conto (see above) which had not only knobs to enable numbers to be input with a quick turn (rather than requiring stylus input), but also an efficient carry and clearing mechanism.

As they became cheaper and more accessible not only to shopkeepers but ordinary citizens who might value the assistance they offered, for example in their role as taxpayers,[6] these devices became increasingly popular. However, they suffered from the same problems of clumsy input and slow operation that afflicted the simple linear machines. Some such as the Adall could only add, others had no provision for zeroing, and all were slow to use. Further, such adders, whilst simple in concept and cheap to manufacture, were so slow to use for repetitive calculation that they were virtually useless for the key arithmetic operations of multiplication and division. A quick method for repeated addition and subtraction was required at minimum to achieve

6. See for example, A.P. Kidwell, "'Yours for Improvement'—The Adding Machines of Chicago, 1884–1930", *IEEE Annals of the History of Computing*, July-September, 2001, pp. 3–6.

these other two functions. That required a more expensive approach and a much more elaborate machine, the first commercial variant of which was designed and sold as the "Arithmometer" by Thomas de Colmar.

4.1.3 Commercial "four function" calculating machines

Arithmometers

Thomas de Colmar first made public his design for a calculating machine capable of doing all four functions (addition, subtraction, multiplication and division) when he was granted a patent for it in 1820.[7] His initial design was very different from the machines that would ultimately become the first successfully commercialised four function calculators, half a century later.[8] The first design consisted of four step drums (along the lines invented by Leibniz, but not necessarily based on having ever heard of, let alone seen the internal mechanism of Leibniz's invention).[9]

Whether or not Thomas reinvented the step drum, even in this first design, the four integers to be added (or subtracted) were set by sliding four corresponding cogs to the appropriate position on such drums. The cogs were moved by sliding nobs along four (square section) axles which were in turn attached to output dial wheels which showed the correct number through small windows. A carry mechanism was incorporated. Rather than being turned by a crank handle the stepped drums were turned by a ribbon that was pulled out from the side. All later designs used a crank handle to turn the drums as is shown in the diagram by Franz Reuleaux in 1862 shown in figure 4.6 below.

The diagram shows a crank handle on the right top which drives the rotation of the step drum (shown centre right). In front of the crank handle can be seen a "slider nob" which moves the cog along the axle to engage the correct number of teeth of the step drum (corresponding to the number to be added). To the left is a reversing mechanism connected to an output dial. The reversing mechanism allows the output dial to be rotated in the opposite direction, if the nob that activates it (situated above) is shifted from the "addition" to the "subtraction" position (where an internal gear reverses the internal rotational motion).

Immediately below in figure 4.7 is a picture of a step drum from a later arithmometer, but based on the Thomas mechanism, showing the slider, drum with its 'counting gear' positioned for the input of '5'. In this arithmometer the input number selected shows in the immediately adjacent window. Note the square section axle on which

7. Patent no. 1420, 18 November 1820
8. For the most authoritative website on the Thomas de Colmar machines and history see Valéry Monier's magisterial site `http://www.arithmometre.org`, viewed 20 July 2013.
9. see Friedrich W. Kistermann, "When Could Anyone Have Seen Leibniz's Stepped Wheel?", *IEEE Annals of the History of Computing*, Volume 21, Number 2, 1999, pp. 68–72. However, as Stephen Johnston (see reference below) points out, images of Leibniz's machine were available from the late C18, see for example 1744 engraving in Annegret Kehrbaum and Bernhard Korte, *Calculi: Bilder des Rechnens einst und heute (Images of Computing in Olden and Modern Times)*, Opladen, 1995, p. 61.

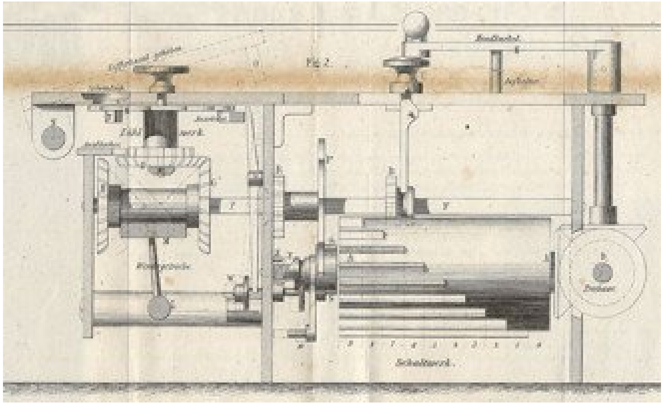


Figure 4.6. 1862: Thomas de Colmar arithmometer mechanism diagram.[10] (Source: Museum of the History of Science, University of Oxford.)

the counting gear moves. This allows the counting gear to engage the axle whatever its position along it.

Figure 4.7. Step drum (in a later 'TIM' arithmometer) (collection Calculant)

The Thomas arithmometer was arranged so that the input mechanism could be shifted in relation to the output dials. In this way it was possible to carry out "long multiplications" (by repeated additions) or "long division" (by repeated subtraction). However, in the early decades the machine was not particularly reliable. In particular, the Thomas arithmometer did not really have a reliable carry mechanism until a patented mechanism was introduced in 1865, and this remained the fundamental system for carry in successive generations of arithmometers for the next 50 years.

As with previous calculating machines, commercial success for the arithmometer machine was far from assured. The high finish in lacquered brass and ivory, heavy brass mechanism in the arithmometer's design still reflected the high status market it was initially seen as appealing to. Thus, it could have become just another prestigious mathematical machine in the cabinets of curiosities of those of elevated status. The somewhat circuitous route to its later success is described by Stephen Johnston.[11] As Johnston notes, Thomas did not himself make these early machines. The first of his machines to survive was made by in Paris by Devrine, as usual a local clock (and instrument) maker.

Thomas de Colmar himself was the director of an insurance company. In the 1820s the insurance industry was rapidly expanding as a consequence of free trade and the transport revolution, and France had become an international centre of the industry.[12] The likely consequent intensification of work no doubt galvanised Thomas's interest in finding an efficient way of handling multiple calculations. But given his responsibilities in an intense commercial environment is unlikely that he would have been able to spend all or even a great deal of his time on designing and building more arithmometers over the next several decades. Indeed it was not until several decades later that, in 1844, the arithmometer, very much re-designed, appeared at a French national exhibition. There it could be found amongst precision instruments in a category of 'diverse measures, counters and calculating machines'.[13] Johnston thus argues that far from being a lone pioneer, the machines of Thomas were in competition (and regarded initially as a poor second) to others. However, over a half century, the machines of Thomas, as they evolved, prevailed.[14] Johnston notes that to achieve this "Thomas campaigned both through the press and in commissioned publications. He also engaged in the rituals of patronage, rituals that we might more readily associate with the decoratively elaborate calculating machines of the 18th century."[15] However, the design of the Thomas machine was sufficiently beautifully executed to make it an appropriate gift to persons of high standing, a property essential to its successful promotion.

Indeed the Thomas arithmometer did take off as a consumer product in France, the UK and Europe and variants continued to sell right up to the first world war. They were not cheap. For example, in 1872 British engineer Henry Brunel wrote that "I have just got what my mother irreverently calls 'a new toy' – to wit a calculating machine price £12 which does all the common operations of arithmetic viz addition, multiplication, subtraction & division in the twinkling of an eye. It is really a very useful article worth its weight in brass."[16] In terms of today's purchasing power, £12 from 1872 was the equivalent of £5,840 (~US$8,900) in 2013 (based on average earnings).[17]

Not surprisingly, the commercial background of Thomas de Colmar gave him a head start in knowing how to manage, promote, and sell his product, and that,

11. Stephen Johnston, "Making the arithmometer count", *Bulletin of the Scientific Instrument Society*, Volume 52, 1997, pp. 12–21.
12. Peter Borscheid, "Europe: Overview", in Peter Borscheid and Niels Vigo Haueter (eds), *World Insurance: The Evolution of a Global Risk Network*, Oxford University Press, Oxford, 2012, p. 39.
13. ibid.
14. As cited by Johnston, Jean Marguin, *Histoire des instruments et machines à calculer*, Paris, 1994, p. 111: 'Pendant un demi-siècle, la machine y régna seule.'
15. Johnston, "Making the arithmometer count".
16. Brunel to Adams, 18 March 1866, Bristol University Library, Brunel Collection, Letter Book VII, f. 106, cited in Johnston, ibid.
17. Calculated using the `http://www.measuringworth.com` calculator on 21 July 2013

must in part be the clue to its success. By the turn of the century Thomas de Colmar arithmometers found homes in science laboratories (particularly astronomical observatories), insurance and engineering companies, and government departments (especially those dealing with finances).

By the time Thomas died, in May 1870, some 800 arithmometers had been made. His son, Thomas de Bojano then took over manufacturing. Thomas de Bojano died in 1881 and Thomas's grandson, the Compte de Ronseray, continued manufacturing arithmometers under the management of Payen. By 1878 some 1500 Thomas arithmometers had been constructed. Some 180 Thomas arithmometers are known to have survived to the present, of which 110 are in public collections and 50 in private collections. Below in figure 4.8 is the Thomas Arithmometer in this collection, which is from 1884.

Figure 4.8. 1884: Thomas de Colmar Arithmometer
Serial 2083 Model T1878 B
(collection Calculant)

The arithmometer in this collection demonstrates the high level of usability that the firm of Thomas had now achieved. It allows the multiplication of two numbers each as large as 9 million and the result can be read out to 16 places. Ivory markers are provided for the decimal point (and to delineate hundreds, thousands and millions for ease of use). The clearing and carry mechanisms are entirely reliable. Indeed, still bearing its original varnish, this particular machine operates reliably some 125 years after it was made. Indicating that its owner was serious about its use in practice the machine appears to have been re-mounted by the famous instrument maker, Stanley, (sometime after 1900) in a better quality and stronger box fixed to a cast iron frame. This allows the machine to be tilted at a suitable angle for easy use. As the only surviving Thomas known to have this improvement, this machine seems to stand as a precursor to later innovations under Payen when he added a (rather less robust) hinged tilt base to his machines.

The repairs carried out on this machine were not uncharacteristic of what was required to keep the arithmometer in reliable working order. The machines however were subject to breakage and expensive to repair. The market for such an expensive machine was quite limited, and despite its uprecedented success, as Johnston concludes, Thomas's work on the machine still fell "more into the category of vanity publishing than mass production".[18] Nevertheless, it formed the standard against which improvements were sought and new designs contemplated.

The time was ripe for others to attempt to produce improved machines. The Thomas concept was developed and improved by a number of other engineers and marketed from different countries.[19] From 1880, other European manufacturers - Burkhardt, Layton, Saxonia, Egli, Bunzel, etc entered the market. The three arithmometers from this collection, shown below in table 4.4, *Arithmometers*, are products of this now enlarging set of competing manufacturers and designers.

The three arithmometers (figure 4.4, *Arithmometers*, in this collection demonstrate the sorts of improvements to the basic arithmometer design which now emerged. The first, a TIM ("Time is Money") arithmometer manufactured by Ludwig Spitz and Co. of Berlin-Tempelhof in 1909 was greatly strengthened from the Thomas design to create a much more robust and reliable machine. First produced in a wooden box it was later made much stronger by installing it in a cast iron frame which held all parts rigidly.[20] Beginning a change in style that would be emulated in many other machines the brass panels were now enamelled black consistent with the management, industrial and commercial settings where it was intended to find its market. The other two machines in this set would also originally have had a similar black finish. (Unfortunately, there was a tendency in the late C20 to "take the machines back to brass" in the same way that in the 1960s there was a trend to remove the original finish from wood on vintage furniture, and plaster from brick walls in old houses.)

Burkhardt in Austria produced the first Austrian version of the Thomas machine, and later improvements were marketed in association with Bunzel. Hugo Bunzel, a former caligraphy teacher in Prague, designed the machines that became known as the Bunzel-Delton. The were manufactured by the Bunzel- Delton-Werk Fabrik automatischer Schreib-und Kechenmaschinen in Vienna.[21] The second machine above is of interest since it is a one of its kind prototype made by these workshops. It

18. ibid
19. P. A. Kidwell, "American Scientists and Calculating Machines- From Novelty to Commonplace", Annals of the History of Computing, Volume 12, Number 1, 1990, pp. 31
20. See Ernst Martin, *The Calculating Machines (Die Rechenmaschinen): Their History and Development*, ed. and trs. by Peggy Aldridge Kidwell and Michael R. Williams, MIT Press, Mass., 1992, p. 191–4
21. ibid. p 198.

Table 4.4. Arithmometers

Date	Description	Maker	Type	Device
1909	TIM (Time is Money) Arithmometer	Ludwig Spitz & Co	Arithmometer	
~1913	Bunzel-Denton Arithmometer	Bunzel	Prototype-Front Crank	
~1917	Madas IX Maxima	Madas	Auto Division	
(collection Calculant - all above)				

shows the “arms race” that was now underway between competitive designs. Already another radically different design of calculator (the pinwheel described below) had emerged to compete with the arithmometer. Its crank handle was mounted on the side of the machine, making it much easier to turn than the top mounted handle of the traditional arithmometer. The second arithmometer in this collection above seems to encapsulate a *patent*[22] filed by Bunzel on May 25, 1914, for enabling the position of the crank to be shifted from its normal position standing upright on the deck to one where it is horizontal on the side of the machine mimicking the convenience presented by the pinwheel. Indicative though this is, the prototype was never put into production.

22. `worldwide.espacenet.com/publicationDetails/mosaics;jsessionid=344FC5328A2BB5BB997876E83E30C740.espacenet_levelx_prod_0?CC=AT&NR=64952B&KC=B&FT=D&date=19140525&DB=&&locale=en_EP`

A much sought after goal for calculator designers was to develop a method which would completely automatically carry out division. All the previous arithmometers required a process of “long division” where the divisor was subtracted until no more subtractions were possible. However, this required the operator to either guess when this point had been reached, or subtract one time too many (creating an “overflow”) and then reversing the subtraction to get the correct outcome. A race had been on to try to solve this particular problem, and in 1901 Alexander Rechnitzer filed a patent in the US Patent Office with a solution. In essence it involved a mechanism that could determine that an overflow had occurred (indicated by the most extreme left hand dial going from 0 to 9). When this occurred the mechanism advanced the carriage one place and added the subtracted number back one time, thus setting the machine up for the next partial division to take place.[23]

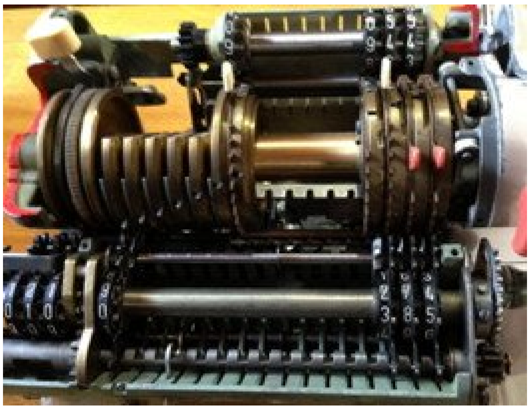
The above principle was incorporated into a MADAS arithmometer, developed by Erwin Janz and manufactured by Egli in Zurich, starting in 1913. The acronym MADAS was short for "Multiplication, Addition, Division - Automatically, Substraction"). The achievement of an entirely automatic division process - the first ever for an arithmometer style calculator - was a considerable triumph. In this collection there is a MADAS IX Maxima calculator (produced in 1917), an ambitious machine also in that it could accept 9 digits, up to one thousand million, as input and could display 16 digits (representing numbers up to ten thousand trillion) in its results register. Two MADAS models were produced and by 1920 about 1,000 were being produced with some 7,000 having been sold from countries as wide apart as Europe and Australia.[24]

Pinwheels

The pinwheel calculator was developed as an improvement on the arithmometer more or less simultaneously in Russia in 1878 by Swedish engineer Willgodt Theophil Odhner (1845–1905) and in the US in 1874, by Frank Stephen Baldwin (1838–1925). Although they worked independently of each other their machines were so similar that the ‘Ohdner’ or ‘Baldwin’ machine became more or less interchangeable terms..[25] The designs utilised a clever ’counting gear’ in which the number of teeth protruding could be adjusted by sliders (and later a push-down keyboard). A pinwheel and the internal mechanism (with some pinwheels removed) from a Walther pinwheel calculator in this collection is shown in table 4.5, *Demonstration Pinwheel Calculator*, below.

23. US Patent 809075, Filed by Alexander Rechnitzer of Viennea, US Patent Office, 29 June 1901, granted 2 January 1906, page 9, clauses 80–110. For a more comprehensive set of his patents see `http://www.rechnerlexikon.de/en/wiki.phtml?srch=rechnitzer&title=Spezial%3APatentpage` viewed 25 July 2013.
24. Martin Reese, Hamburg, *55 erfolgreiche Jahre: MADAS-Rechenautomaten aus der Schweiz 1913 - 1968*, `http://www.rechnerlexikon.de/files/MADAS--8-2010.pdf`, viewed 18 February 2012.
25. Diana H. Hook and Jeremy M. Norman, *Origins of Cyberspace*, Novato, California, 2002, p. 255.

Table 4.5. Demonstration Pinwheel Calculator

	
A single pinwheel	Mechanism of the Walther pinwheel calculator (parts removed for demonstration)
(collection Calculant)	(collection Calculant)

As can be seen above, the pinwheel (left, above) is a thin disk with the slider shown protruding in the top right corner. It is in the '3' position, with the cam slot rotated pushing 3 teeth out (seen between and behind the fixed teeth at the bottom). These then are the teeth that are added by the counting wheel.

These pinwheels thus replaced the much larger and more cumbersome step drums of the arithmometers. Thin as these were, and able to be nestled side by side in a compact manner (right, above), these allowed the calculators to be lighter and much more compact than the arithmometers. The crank handle was conveniently on the right end of the machine, and could be turned either forward or backwards (corresponding to addition or subtraction). The output wheels were set in the base, and as with the arithmometer, the input mechanism (including the pinwheels and sliders) could be moved parallel to it allowing long multiplication and division to be carried out.

Ohdner began manufacture of his machines in 1886 under the name "Original Ohdner" but sold his rights also to Grimme, Natalis and Co., Braunsweig, who began marketing the machines under the name Brunsviga.[26] In turn they also licenced agents such as Schuster to sell them. Reflecting a later start these machines were made from a wider range of materials than the arithmometers, including iron and nickel alloys as well as brass and steel. The Brunsviga was a quick success selling 20,000 units between 1892 and 1912.[[27] Below in figure 4.9 is the very early Brunsviga-Schuster pinwheel calculator from 1896, followed in table 4.6, *Pinwheel Calculators*, by the other pinwheel calculators in this collection.

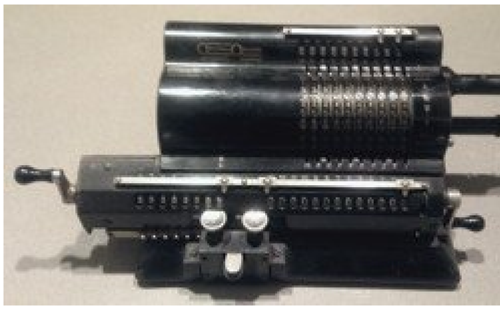
26. ibid
27. Hook and Norman, *Origins of Cyberspace*, p. 255.

Figure 4.9. ~1896: Brunsviga Schuster Pinwheel Calculator
Serial 3406
(collection Calculant)

Table 4.6. Pinwheel Calculators

Date	Description	Maker	Type	Device (click for greater detail)
1945	Facit Model S Calculator serial 210652	Facit	Pinwheel	

Continues…

~1951	Original Ohdner Model 39 serial 39–288965	Ohdner	Pinwheel	
~1957	Walther WSR160 Pinwheel Calculator	Walther	Pinwheel	
~1957	Walther WSR160 Demonstration Pinwheel Calculator	Walther	DemonstrationPinwheel	

(collection Calculant - all above)

As the range of dates for the set of pinwheel calculators in this collection suggests, the pinwheel design was highly successful and capable of being improved on for the better part of three quarters of a century. The second Ohdner in the collection is from 1938, and although possessing some improvements (such as the capacity to move the carriage back and forth with tab keys), was little different in concept. The same can be said for the Facit calculator from around 1945, and even from the 1950s, near the end of the production of such machines, despite its more modern finish and use of plastic fittings (and some improvements such as clearing levers), the Walther 160 remains quite similar in operation to the Brunsviga of 1896, which, significantly, still works as smoothly as the Walther.

The main progress that had been made was thus not so much in functionality as in production methodology. This is epitomised by the Walther 160 and later Walther calculators. Karl Walther's ancestors made rifles and in 1858 he established his own hunting and shooting rifle company..[28] He developed then into pistols including the famous Walther PP series military pistols, production of which began in 1928. They proved to be a worldwide success. However, the second world war in 1945 left his son Fritz with 80 patents and little else. He was able to rebuild the business both in relation to weapons, but also by diversifying into making office machinery, and in particular calculators. By the 1950s he had four large factories equipped with advanced machinery and technique.[29] The Walther 160 was by now a classic product of modern manufacturing technology, mass produced, comparatively lightly made, and accessibly cheap. As a result many thousands of them were sold. By 1970 the Walther Office Machines company (Walther Büromaschinen GmbH) employed 2000 staff and was producing almost 120,000 machines per year, with about 50% for export.[30]

There were of course many variations on this theme. These included the "Frieden Wheel" which managed to achieve the same effect as the pinwheel but through simply a nicely timed use of a cam. More of that sort of mechanical detail for all the machines referred to here can be found in the classic book by Martin,[31] and the marvellous websites of Rechnerlexikon[32] and John Wolff.[33]

Proportional Rack Calculators

One helpful development in design was the development of a proportional rack which moved cogs through a series of increments depending on the length of a toothed rack with which they engaged. Shown below in figure 4.10 is the rack mechanism of such a calculator. Note how successive racks have moved increasing distances as the crank handle is turned (creating the diagonal pattern). The loose cogs seen on the square section axle are an example of the nine such cogs under each column of keys. Normally lying between the racks when a key is depressed it moves the corresponding cog sideways on the axle to engage with the appropriate rack for the number of the key depressed.

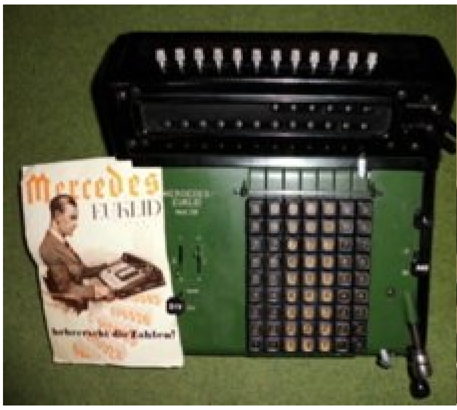
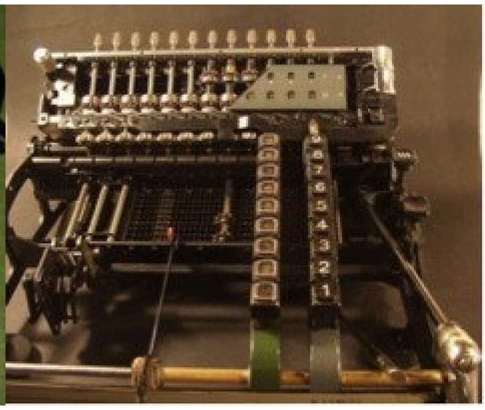
In 1906 Christel Haman founded the Mercedes-Euklid company who adopted the rack mechanism. In 1910 he incorporated the mechanism patented by Alexander Rechnitzer, referred to earlier, which enabled his machine to automatically perform the process of division, thus becoming the first mass produced machine with this

28. "Walther Company History", `http://www.carl-walther.de/cw.php?lang=en&content=history`, viewed 23 July 2013.
29. Ray Mackay, "The Walther Company: Historic Recollections", December 1997. `http://www.xnumber.com/xnumber/walther.htm`, viewed 22 July 2013.
30. John Wolff, "John Wolff's Web Museum: Walther Calculators Overview" `http://home.vicnet.net.au/~wolff/calculators/Walther/Walther.htm`, viewed 23 July 2013.
31. Martin, *The Calculating Machines*
32. `http://www.rechnerlexikon.de/en/artikel/Spezial:Allpages`
33. `http://home.vicnet.net.au/~wolff/calculators/`

Figure 4.10. Rack mechanism
of a Mercedes-Euklid 29 calculator.
(collection Calculant)

capacity. The fully working Mercedes-Euklid 29 can be seen below (left) and a demonstration Mercedes-Euklid (model 29 from 1934) can be seen below (right) in table 4.7, *Proportional Rack Calculators*.

Table 4.7. Proportional Rack Calculators

	
1923: Mercedes-Euklid Model 29 calculator (collection Calculant)	1923: Mercedes-Euklid Model 29 Demonstration calculator (collection Calculant)

Comptometers

A further approach to the problem of the four functions is found in the American invention by Felt, of a device that he called the comptometer. He patented it in 1887. It performs the task of addition by a system of keys and levers. The keys press down rods which, from key 1 to key 9, increase incrementally in length for each successive key. This difference in displacement, magnified by a lever turns an accumulating gear through 1 to 9 teeth as appropriate.

Notably no crank handle is needed. The small force required for the addition comes simply from the key press. The process of multiplication is done by multiple presses of the appropriate key. This is fast, not the least because with two hands up to ten keys can be pressed at once. But subtraction must be carried out by addition of complementary numbers. Carrying of "tens" is implemented between the accumulating read-out wheels.

Felt first demonstrated this famously with a model constructed in a wooden macaroni box in 1885. He patented his design in 1887 and began selling it from his manufacturing company in Chicago. In 1890 he gave it the name “the Comptometer” and energetically promoted it.[34] Below in figure 4.11 is the Comptometer in this collection - an example from 1896 of the first model, cased in wood, and one of the 40 oldest known to still be in existence. Remarkably with a drop of oil on the springs it still works perfectly.

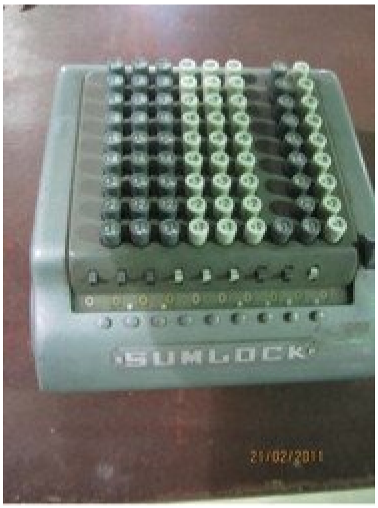
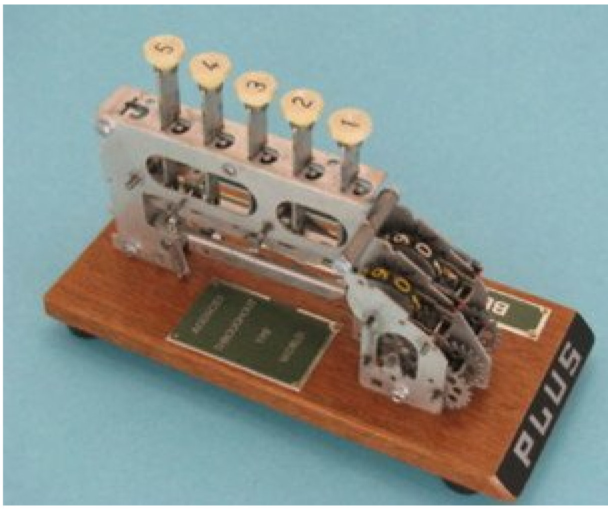
Figure 4.11. 1896: Felt and Tarrant Comptometer: earliest wood-cased model, serial 2491 (collection Calculant)

The comptometer was a highly successful innovation and was sufficiently cheap and useful to reach a broad market. Characterised by extremely effective marketing and capable of being mass produced the device continued to be marketed for some 80 years. Unusually for such innovations it left its inventor extremely wealthy. By then its essential mechanism was being utilised by more than one company. Shown below

34. ‘/Origins of Cyberspace, p. 244.

in table 4.8, *Comptometers*, is the remaining comptometer in this collection, this time made of plastic in the 1950s by the British Bell Punch company. Also shown is a demonstration model showing the keys and lever system used in the Bell Punch machine.

Table 4.8. Comptometers

Date	Description	Maker	Type	Device
1950s	Bell Sumlock Comptometer	Bell Sumlock	Model 909/S/117.878	
~1955	Bell Punch Sumlock	Bell Punch	Demonstration Comptometer	
(collection Calculant - all above)				

Curta: The peak of miniaturisation (1947–1970)

Finally there is the last and most beautifully miniaturised of the four function manual mechanical calculators. They were designed by Curt Herzstark. Curt was born in 1902 as the son of Samuel Herzstark who had established a calculator manufacturing company in Vienna in 1905–6. Curt's father died in 1937 but by then Curt was both Director of the company and a highly competent designer in his own right. He recognised the need for a miniaturised four function calculator which could be carried in an engineer's pocket. He began to design this as a cylindrical calculator which could be held in one hand and operated with the other. It could utilise the modern new light weight alloys of aluminium, magnesium, etc. By the end of 1937 he had the form of what he wished to accomplish firmly in his mind. It would be compact, lightweight, and in order to allow a carry mechanism which would allow the crank to be turned only one way, it would use an internal process of complementary arithmetic for subtraction. Then came Hitler. In 1943 Herzstark found himself arrested and imprisoned in the Buchenwald concentration camp. There his engineering skill was noticed and he was placed in a small technical camp to manufacture components to assist the Nazi war effort (in particular, parts of the V I and II missile bombs).

While Curt Herzstark had been imprisoned at Buchenwald, the Germans had retreated from Italy and whilst doing so had seized some office machines of which two truckloads were delivered to the camp. After Curt unloaded them one of the local factory owners came over to inspect them. He turned out to be Fritz Walther, the son of Curt's father's competitor. The Walther company was now back to making weapons for the war effort. But Walther recognised Curt Herzstark and later told the Camp Commandant of Curt's high skills and background. Soon after his supervisor, Herr. Munich, called Curt over. Curt recounts the conversation as:

> Mr. Munich said: "See, Herzstark, I understand you've been working on a new thing, a small calculating machine. Do you know, I can give you a tip. We will allow you to make and draw everything. If it is really worth something, then we will give it to the Fuhrer as a present after we win the war. Then, surely, you will be made an Aryan." For me, that was the first time I thought to myself, my God, if you do this, you can extend your life. And then and there I started to draw the CURTA, the way I had imagined it.[35]

Curt Herzstark's role gave him a relatively protected status and he survived to 1944 when the camp was liberated by US troops. His calculator plans drawn up in pencil, complete with all dimensions and tolerances were completed just as the war ended. He developed the prototypes with the Rheinmettalwerk typewriter and calculator factory which was still operating near Weimar where he was named a Director. However for a range of reasons after much exploration he agreed to a proposal by the Prince of Liechtenstein to produce the calculators there at a company established for the

35. *An Interview with Curt Herzstark*, OH 140, conducted by Erwin Tomash on 10–11 September 1987, Nendeln, Liechtenstein, (English Translation).

purpose which was named Contina AG. The financial arrangements did not live up to their promise, but because he owned the patents he was able to negotiate what was in the end a satisfactory outcome.

The first model (the Model 1) began production in 1947. Below left in table 4.9, *Curta Calculators*, is a (rare) example of the Model I Curta. It still has the pin sliders which were soon to be improved upon and is in mint condition. It was made in ~July 1948, the year after production began, and is in the first 5,500 made. As can be seen the sliders are pins which were used by Curta before he introduced bakelite handles on them. Centre is a later Curta from 1967 complete with the original box in which it was sold and its instructions, all in mint condition. On the right is a Curta Model II from 1962. The Model II was a larger machine capable of a number input to 11 significant figures, compared with the more compact Model I which could accept a number accurate to 8 significant figures. Note the very ‘modern’ anodised aluminium sliders on the Model II which comes also with an optional leather carry case (not shown here).

Table 4.9. Curta Calculators

		
~July 1948: CURTA Type 1 (pin sliders) earliest model calculator serial 5424 (collection Calculant)	1967: CURTA Type I calculator serial 76436 & cardboard box (collection Calculant)	1963: CURTA Type II calculator (collection Calculant)

On the Curta above, left we see the clearing ring protruding. Bringing all numbers to zero is achieved by lifting the top section and turning the clearing ring. Adding is achieved by setting successive numbers and turning the crank through a full turn. Ingeniously, pulling the crank up sets it for subtraction. Multiplication is done in the usual manner of multiplying by each successive integer of the multiplier with the upper register being advanced for each one by lifting the top section and rotating it one notch. Division is by the usual method of long division by repeated subtractions. Once more, for each partial product the operator must either correctly judge the correct number of times to turn the crank, or if an overflow is forced, reverse the operation by pushing in the crank and turning it one full turn. In short it is an excellent machine for addition and subtraction, but involves the normal acquired skill, and tedium, required by multiplication and division in any of the mechanical calculators not equipped to do it automatically.

These machines thus constitute the pinacle reached in the development of the personal hand-operated mechanical digital calculator - able to carry out all four functions by means of a crank operated machine of exquisite miniaturised workmanship and design. Production ceased in November 1970 although sales continued through 1973. At least 150,000 of the various calculator models were made.[36]

4.1.4 The quest for direct multiplication and division

An entirely different evolutionary path (mentioned earlier) attempted to solving the problem of mechanising the four arithmetic operations (+, -, x, /). The emphasis was on finding ways to directly perform the more difficult two operations of multiplication and division. The approach was a development from Napier's rods - or "bones" (developed by John Napier (1550–1617). As already mentioned calculational approaches had been designed around these principles by Wilhelm Schickard in 1623, Charles Cotterel in 1667, Gaspard Schott in 1668, and Samuel Morland between 1625 and 1695.

The earliest of these machines had been that of Schickard (below, left). In the upper part is a set of rotatable Napier's rods revealed by windows to give partial products. In the lower part is the world's first known stylus operated adding machine to add these partial products up. A late and unique expression of these in this collection (below, right) is Justin Bamberger's Omega Calculating Machine (1903–6). In the upper section is a set of Napier's bones revealed by moveable windows for discovering the partial products of two multiplied numbers. In the lower section is a Locke adding machine for adding them up.

The earliest of these machines had been that of Schickard (figure 4.12) which was invented in 1623. As described earlier, it used a set of rotatable Napier's rods in its upper part to yield partial products of the multiplication of two numbers, whilst in its lower part was the world's first known stylus operated adding machine which could be used to add the partial products up.in

A later and unique expression of the same principles can be found in this collection (shown in figure 4.13). It is Justin Bamberger's Omega Calculating Machine (1903–6). In the upper section is a set of Napier's bones revealed by moveable windows for discovering the partial products of two multiplied numbers. In the lower section is a Locke adding machine for adding them up.

36. *An Interview with Curt Herzstark/]* For more on the Curta see the marvellous collection of documents and simulations at *'/The Curta Caclulator Page* .[/The CURTA Calculator Page/', `http://www.vcalc.net/cu.htm`, viewed 25 July 2013.

Figure 4.12. 1623: Recreation of Wilhelm Schickard's calculating machine (collection Calculant)

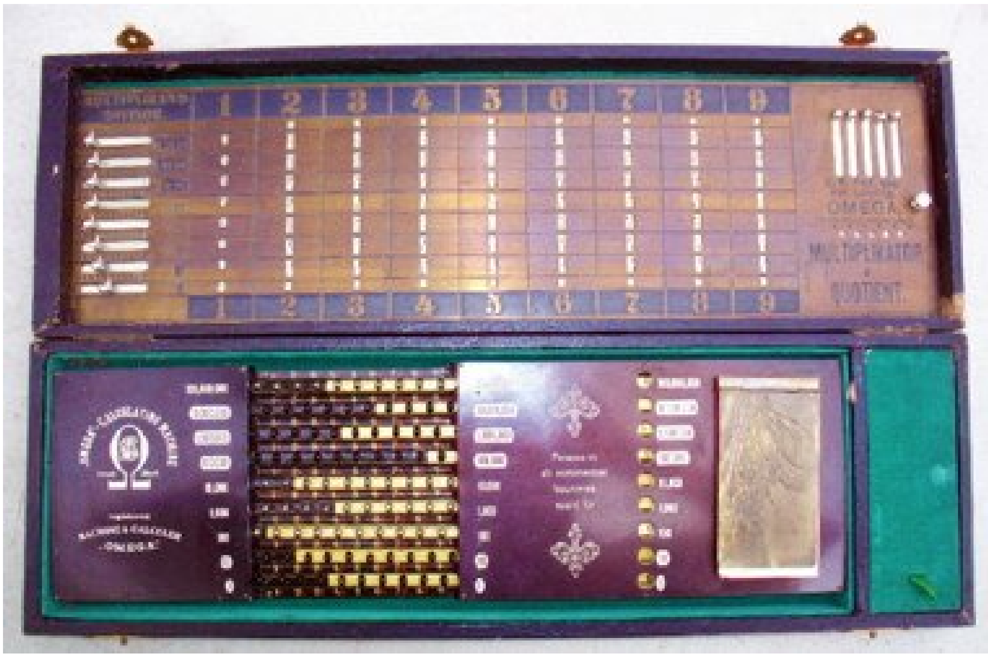


Figure 4.13. 1904–1905: Bamberger's Omega calculating machine (collection Calculant)

Bamberger's Omega used linear strips rather than the rotatable mechanisms in the upper and lower sections of the Schickard calculator. It also has some additional provision for storing intermediate results to assist long division, including the register on the top right, and the notebook. Otherwise the two machines are very similar in operation and both, with some considerable effort, can be used to perform all four functions of arithmetic. The fact that neither took off in the market place may be in part a factor of their difficulty of use and part a lack of adequately determined marketing.

A much heavier and complex mechanical approach was also explored. First it was embodied in Léon Bollée's calculating machine which won a gold medal at the Paris Exposition of 1889. One surviving example of this bulky but beautiful machine can be seen at the Musée des Arts et Métiers in Paris. This collection has only an article on this "New Calculating Machine of very General Applicability" from the Manufacturer and Builder of 1890, see figure 4.14 below.

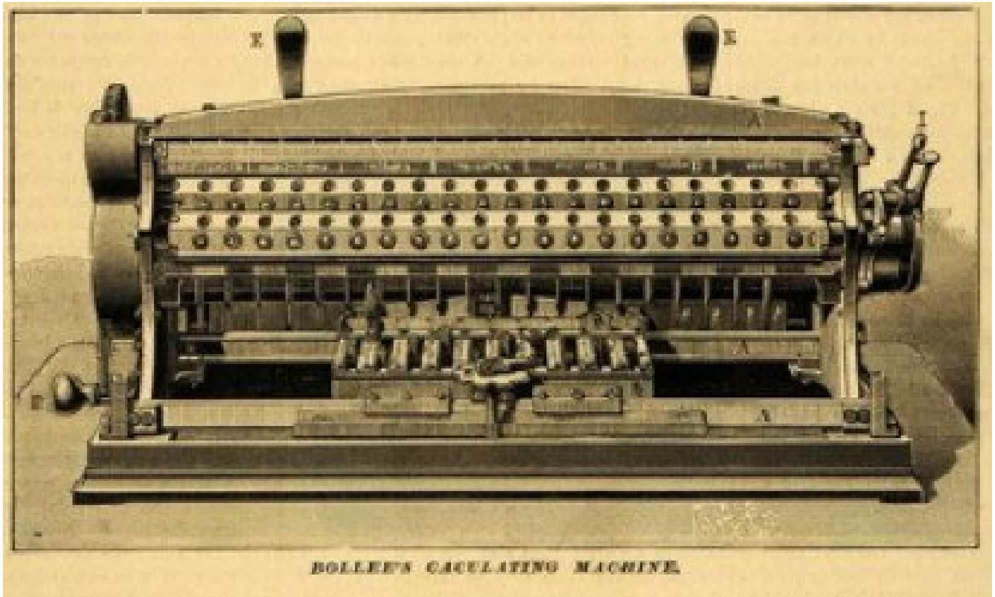


Figure 4.14. Leon Bollée Calculating Machine
"A New Calculating Machine of very General Applicability"
The Manufacturer and Builder 1890[37]
(collection Calculant)

Similar principles were however utilised by Otto Steiger in Switzerland who in 1895 patented a rather more practical "Millionaire calculating machine" which had a simple enough mechanism to enable production on a commercial scale (see figure 4.15below).

The Millionaire calculating machine combines the idea of the physical embodiment of multiplication tables with that of the proportional rack. It is able to interrogate a multiplication table represented by metal rods, and in a single crank of the handle multiplies the multiplicand by a number set between 1 and 9 then advancing its

Figure 4.15. 1912: Millionaire Calculating Machine, serial 2015 (10×10×20) (collection Calculant)

internal carriage one place ready for the next multiplier to be set and applied. Thus, for example, to multiply 4689 x 2568 an arithmometer or pinwheel would take 21 cranks of the handle (8+6+5+2) whereas the Millionaire could achieve the same outcome with only four cranks of the handle. In the lid was a set of tables of factors to assist division, a brush to keep the machine clean, and a special bolt so when being transported the carriage was held clamped in place, since if the machine were dropped the carriage was heavy enough to punch through the end of the case.

The deck of the Millionaire in this collection is shown below in figure 4.16, *Millionaire deck*, with the selector on the left which picks the multiplying factor, and the selector on the right which sets it for addition, subtraction, multiplication and division (AMDS). The ten sliders for setting the number to be operated on are obvious, as are the result windows, and on the far right, the crank handle.

Manufactured by H.W. Egli, some 4,655 Millionaires were sold between 1895 and 1935[38] at a 1912 price of about US$480[39] (about $11,700 in 2013 US dollars[40]). The Millionaire calculating machine in this collection was manufactured on 16 October 1912,[41] and was until 1954 held by the B. B. Company in New York, NY. It is rather

38. *Origins of Cyberspace* p. 242.
39. Price list in Edmonds Collins, *The Millionaire Calculating Machine*, pamphlet, Edmond Collins, 35 Dearborn St, Central Chicago, ~1912. See also price of US$500 at 1914 prices given by Luc de Brabandere cited in B.O.B. Williams," Check Figures—A Once Ubiquitous Tool for Book-keepers, published in *Slide Rule and Calculation monographs*, Slide Rule Circle, UK, 2002, pp. 59–98.
40. Calculated using the `http://www.measuringworth.com` calculator on 24 July 2013
41. Letter from H.W. Egli Ltd, Zurich, dated 20 November 1967 (collection Calculant).

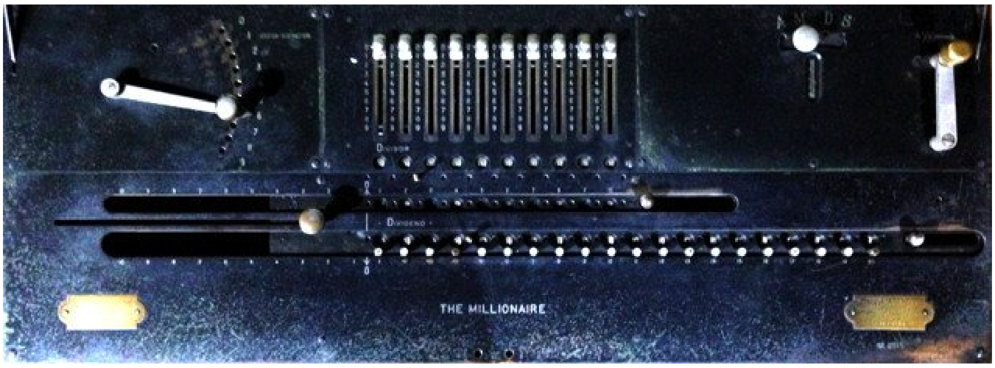


Figure 4.16. Deck of the Millionaire Calculating Machine (collection Calculant)

rare since it is capable of greater accuracy - 10 column - than the more common 8 column ones. The machine still operates reliably after more than 100 years.

The Millionaire was reliable, but heavy (37 kg or 81 pounds) and expensive. It could produce an answer to 20 significant figures (100 billion billion). It came in various configurations, including with a keyboard mounted on top to drive the sliders. Its value to the user depended on whether long multiplications and divisions were central to the work to be done. Styled in a rugged 'no nonsense' industrial design it was adopted by scientists who swore by it, railway and telegraph companies, government treasuries, and other technically oriented companies and agencies.

4.1.5 Harnessing electricity

Electricity could be utilised in appliances once it was available through an electricity grid. In the US, the first electrical supply was constructed in 1882 for lighting, with 85 customers. Electrification spread over subsequent decades, primarily in the big cities through private power companies in the first two decades of the C20. In 1926 in the UK separate electricity grids began to be connected into a national grid. It was not surprising therefore that this period of the early C20 was conducive to the introduction of electric motors to many purposes, including adding machines.

The electric motor marked the beginning of the end for all forms of mechanism more ingenious than those depending on the simple minded operation of addition and its inverse, subtraction. The greatest gains in efficiency could be obtained by simply increasing the speed with which these operations were repeated and controlled. Speed gains followed from simpler rather than more complex basic mechanisms. The control mechanisms that utilised these simple repeated basic operations, however did become more complex in the interests of using them to produce more complex and accurate outputs.

Samuel Herzstark (1867–1937), the father of Curt Herzstark who built the Curta, also was a pioneer in calculator construction and together with Gustav Perger established the Austria Calculator Machines Manufacturing Company in Vienna in 1905–6. In an interview in 1987 Curt Herzstark reports that in 1907 Samuel became the first to attach an electric motor to an arithmometer, which he equipped also with a keyboard. Curt also notes that this machine was equipped with automatic division.[42] However after the 1914–18 War Herzstark returned to a demolished business. He restarted with a combination of importing and selling calculators from other manufacturers, assembling old stock of his own design, and then as the business built up designing new machines. Below in figure 4.17 is an arithmometer in this collection branded by Samuel Herzstark from 1929. It is actually a Badenia (manufactured by the German Company Math. Bauerle in the Black Forest) which Herzstark was at that time re-badging and selling.

Figure 4.17. ~1929: "Herzstark" electric Calculating Machine serial 6549
badged by Herzstark, Vienna
(essentially a Badenia Model TE 13 Duplex)
(collection Calculant)

This calculator features its original electric motor, still in good working order, and with a keyboard instead of sliders for input. The features of it were not only that it had a keyboard for input, but also a control mechanism consisting of a column of keys which enabled a number to be entered and then added through 0 to 8 repeats representing multiplication by one to nine. A view of the mechanism of this machine is shown below in figure 4.18.

In the left lower corner of the above can be seen the motor, now coupled to the characteristic step drums of an arithmometer (bottom center and right), with the carry mechanism above. This puts paid to any simple story of the linear development of

42. *An Interview with Curt Herzstark*, OH 140, conducted by Erwin Tomash on 10–11 September 1987, Nendeln, Liechtenstein, (English Translation).

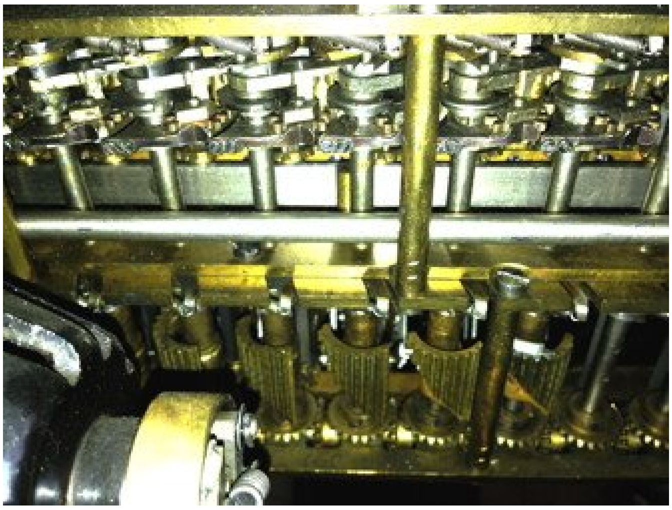


Figure 4.18. 1950s-1960s: Underneath view of the Herzstark mechanism (note the stepped drums)
(collection Calculant)

innovation in the calculator. Here the most modern device of the motor is being coupled to the longest serving commercial system of an arithmometer.

Whilst the above was an obvious innovation, the clumsiest approach in all the calculating devices - from the first arithmometer through to the Millionaire was division, which could only be done by a process along the lines of that done in long division. That is, the number to be divided (the dividend) is considered sequentially from the highest power of ten, and thus decomposed into a series of partial products of the successive parts of the dividend with the divisor.

In the Millionaire the outcome for each partial product could be achieved by setting the correct number in the divisor with the selector. In the Herzstark arithmometer a key column to the far right (black keys, barely visible) allowed the operator to set an addition or subtraction to repeat up to 9 times giving the same effect as with the Millionaire. Thus, although Egli and Co. did fit a motor to their Millionaire, once a motor was available with this rudimentary control mechanism, the advantage posed by the complex and heavy mechanism of the Millionaire was largely lost.

As already mentioned, both the MADAS arithmometer (and Mercedes-Euklid rack calculator) had utilised the invention by Rechnitzer to allow an overflow to be sensed and addressed by the mechanism so that division could be carried out fully automatically. With progressive further modifications, including the insertion of an electric motor, the MADAS became the highly effective electric mechanical calculating machine whose use continued right into the 1960s.

The late MADAS 20BTG calculator, seen above in figure 4.19, which is part of this collection represents a classic exemplar of the late stages of this evolutionary development. Now fully utilising an electric motor it could fluently perform all the

Figure 4.19. 1950s-1960s: MADAS
Model 20BTG serial 94046 electric calculator with true automatic division
(collection Calculant)

operations of arithmetic, complete with automatic clearing and moving the carriage entirely automatically as necessary. It could also be further extended to automatically extract square roots. Its mechanism was also adopted in its essentials, then with many innovations (but with less of the Swiss robust construction of the Madas), in the US Frieden calculators. As it turned out, this final class of machines represented the pinacle of achievement in motorised mechanical calculation.

4.1.6 The Vanishing point: solid state electronics and the arrival of the HP35

Another means of performing arithmetic had largely been neglected for calculators. It had already been employed in the enormous computers that had developed from earlier work by Babbage and Scheutz. They had devised remarkably complex special purpose "difference engines" for calculating logarithms. It had also been utilised in later work by Turing and others giving rise to the electronic computing machines developed in the second world war for decoding. This was the use of binary arithmetic. The electronic version initially used valves to control on/off electric circuits each representing a single binary digit (or bit). 1 was represented by 1, two by 10, three by 11 and so on. It has been recognised since Leibniz that arithmetic could be done with these (since they represented numbers). Indeed - the method is in retrospect obvious. 10 + 01 = 11. 11+01 = 100, etc.

Using switches it was therefore possible to build a very efficient calculating machine. Valves were too bulky, energy consuming, and unreliable for a consumer device but prior sales of mechanical calculators had by now established a massive potential market. The invention of the transistor in 1947 at Bell Telephone labs, based on the quantum properties of crystals, laid the way for "solid state" electric switches at tiny scale, able to be turned on and off by one another. Light emitting diodes (LEDs) - another solid state device which emitted light when electrons having been forced into a higher energy ("excited") state fell back to their stable energy - began to appear as practical output devices in 1962.

The first calculators to use solid-state electronics in desktop form were the ANITA VII and VIII calculators launched simultaneously in 1961, too early to use LEDs, and using instead vacuum tube displays. A variety of desktop four function calculators followed. But these could not replace the portable slide rules and other mathematical aids that were still in use in parallel with the large mechanical desktop calculators that these electronic machines began to replace.

It was in July 1972, that the old ways for scientists and citizen alike were definitively undermined. In that month, to some astonishment, the Hewlett Packard company produced the HP-35 electronic 'pocket' calculator (see in table 4.10, *Hewlett Packard Pocket Scientific Calculators*, below). It was an extraordinary leap forward, equipped not only with a a red LED display, and showing smooth performance of the arithmetic functions but also with provision to calculate reciprocals, powers, square roots, logarithms and anti-logarithms (base e and 10), the trigonometric functions, pi, and a system of registers which enabled chain operations without having to write down intermediate results.

For those of a technical orientation who could afford it, all else was now as obsolete as gaslight. Hewlett Packard followed quickly with the HP-45 appearing in the following year with a configurable display, more functions and registers, a variety of constants, and a more compact shape, more suited for the male shirt top pocket. A year later in 1974 the HP-65 was launched with all that could be done by the HP-45 but with the added feature of being user-programmable through a small built in magnetic strip reader. With that the diminutive HP calculator had taken a huge step towards the first mass-marketed personal computers (the TRS-80 - the author wrote his second book on one of these in 1981 - launched by Tandy, and the Apple-II by Apple, both launched in 1977).

The launch of the HP-35 marks what might be called the "First Vanishing Point" - the point at which all the arithmetic operations and common scientific functions required for calculation became available in a single small electronic calculator, capable of being carried on a belt or in a shirt pocket (albeit in serious peril of it falling out when one bent over to pick something up). Electronic calculators now proliferated throughout the developed and developing world displacing the rigours of mental arithmetic and caught in an 'arms race' to make them smaller, more powerful, easier to use, and find a niche in a highly competitive and very cluttered market.

Forty years later, in 2012, electronic solid state calculators could be found in their billions across the world. By then what might be labelled the "Second Vanishing Point" - the point at which these electronic calculators began to disappear, displaced now by "virtual calculators" encoded in the software of desk and lap-top computers,

Table 4.10. Hewlett Packard Pocket Scientific Calculators

	
July1972 Hewlett Packard HP 35 Calculator serial 1230A 79429, 1972 (collection Calculant)	December 1973 Hewlett Packard HP 45 Calculator serial 1350A 36719, 1973

tablets, smart phones, and much else was coming into in sight (although probably had not yet arrived). But from that vantage point, even the stand-alone electronic calculators, might soon begin to follow their mechanical predecessors of two decades before fading into the misty light of receding memory.

4.1.7 In Lieu of a Conclusion

One feature of this account is it is built around a collection of historical objects. It is reasonable to ask why collect them? Why not just build it based on the great diversity of photographs of these objects in books, journal articles and on the web. The technologies are well described elsewhere in much more mechanically detailed websites. So why bring in the collection?

One reason for collecting is a banal one - the desire to collect and own things of rarity, and of course the thrill of the chase in finding, identifying and acquiring them. But beyond that it is possible for the objects to convey insight. The objects embody stories of innovation which can raise multiple questions. For example, why at some particular moment were these particular objects invented? And at a particular time what was it that allowed some, but far from all of the objects, once invented, to be taken up in use? In particular why did some get established in actual and widespread use? What limited so much more sharply the success of others? Certainly not all the answers to these sorts can be found here. But it is possible to make some potentially useful observations.

Of course, we have not considered even all the available information. There is the story of the detail of the evolution of the mechanism of the bewilderingly wide array of calculators that were built, which at best has been broadly gestured at here. Even so, perhaps enough has been said to indicate that many good ideas have waited around for their moment to be realised rather than simply the whole being driven by discovery. Much could have happened after Schickard and Pascal, but it took centuries for the various devices to become widely useful. Of course, as discussed, the device of Schickard had a potential audience in the restricted group of natural philosophers (in particular astronomers) with whom he communicated. Pascal found his machine more used as a curiosity amongst the aristocracy to add prestige, than used to add up money which was its inspiration.

There is another important consideration in relation to these sorts of innovations. An encounter with these devices suggests that that it is not possible to fully understand them, including their limitations and potential, without actually using them. A trip to, say, one of the great technology museums (for example, CNAM in Paris) will tell you how limited the experience is of, for example, seeing a Pascaline, or reading an essay about it. It is a very different experience to try to calculate with one. This leads to a rather interesting connection between understanding these technologies retrospectively, and the reasons that led to their development in the first place. For the success of that development depended on whether people learned to use them. In short this is a history not only of mechanism but of learning how it can be used.

This leads to the following observation: to learn how to use a calculating technology is not just a matter of understanding its concept. It also requires the acquisition of a type of knowledge which Jean-François Gauvin[43] refers to as "gestural". Gestural knowledge is the embodied knowledge that artisans rely on when they execute work of high craft skill. The first time someone tries to use a Thomas de Colmar arithmometer learning how to add and subtract on it efficiently takes some time. Doing division and multiplication is much more challenging, especially if attempted without a good knowledge of how it can be done on paper. The first time is slow and subject to mistakes. It is only after repeated practice that the gestures required become so practiced that they are quick, instinctive and reliable. It is only then that the potential for calculation with such a device can be can be judged. And it would only have been after this sort of practice that one could credit the Gentleman's Magazine of 1857 claim that:

> M. Thomas's arithmometer may be used without the least trouble or possibility of error, not only for addition, subtraction, multiplication, and division, but also for much more complex operations, such as the

43. Jean-François Gauvin , Habits of Knowledge: Artisans, Savants and Mechanical Devices in Seventeenth-Century French Natural Philosophy, The Department of the History of Science, in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the subject of History of Science, Harvard University, Cambridge, Mass., USA, November 2008, pp. 113.

> extraction of the square root, involution, the resolution of triangles, etc . . . A multiplication of eight figures by eight others is made in eighteen seconds; a division of sixteen figures by eight figures, in twenty four seconds; and in one minute and a quarter one can extract the square root of sixteen figures, and also prove the accuracy of the calculation . . . [44]

As mentioned earlier (in *Part 1* (p 9)), gesture and mind are interconnected. When we 'learn' a gesture, recent neurological research demonstrates that our brains are growing new connections, in a sense rewiring, to accomodate that as what becomes a 'habit'. Therefore, the use of the technology changes our minds. So there is a co-evolution between minds and the combination of the technology and what is required to use it successfully (sometimes referred to as 'technique').

The history of the technology has been one of a competition between old habits, preparedness to learn new technique, and perceived need to do so. It is not easy to throw off old successful technique and replace it with the hard acquired new approaches. Part of the success of a technological innovation is thus likely to depend on the extent to which social forces may encourage through benefit, or require through necessity, the new learning required to use it. Most new inventions are of course promoted with claims that the benefits for user or employer will outstrip the costs of change. Not infrequently these benefits might be initially overstated. Thus for the arithmometer the *Gentleman's Magazine* claimed:

> Instead of simply reproducing man's intelligence the arithmometer relieves that intelligence from the necessity of making the operations. Instead of repeating responses dictated to it, this instrument instantaneously dictates the proper answer to the man who asks it a question. It is not matter producing material effects, but matter which thinks, reflects, reasons, calculates, and executes all the most difficult and complicated arithmetical operations with a rapidity and infallibility which defies all the calculators in the world . . . .. It will soon be considered as indispensable and be as generally used as a clock . . . . [45]

These overblown claims are reminiscent of similar claims for their inventions by Pascal (with this machine the user can do with ease, "without effort of memory" and "without even thinking of it", every possible arithmetic operation[46]) and Moreland (allowing addition and subtraction "without charging the memory or disturbing the

44. *The Gentleman's Magazine*, Volume 202, "The monthly intelligencer", January 1857, p. 100, `http://books.google.fr/books?id=Rf0IAAAAIAAJ&pg=PA100&dq=arithmometer&as_brr=1#v=onepage&q=arithmometer&f=false`, viewed 23 July 2013.
45. ibid
46. Pascal, ligé de retenir ou d'emprunter les nombres nécessaires, et combien d'erreurs se glissent dans ces rétentions et emprunts, à mois d'une très longue habitude et, en outre, d'une attention profonde et qui fatigue l'esprit en peu de temps. Cette machine délivre celui qui opère par elle de cette vexation; il suffit qu'il ait le jugement, elle le relève du défaut de la mémoire; et, sans rien retenir ni emprunter, elle fait d'elle-même ce qu'il désire, sans même qu'il y pense." p. 337

mind[47]). They show the hope, that the mental activity could be replaced by the machine. But in reality, for all of these machines it was not just a question of the high cost of obtaining it, but the learning and practice required to use it.

The Pascaline and Moreland's inventions may have served their inventors in a range of ways, but it was not necessarily to find a broad market for them. The learning required to use it was too great, and the benefit too little in relation to existing technique. As noted earlier, even Thomas de Colmar's arithmometer and its early successors remained on the edge of this balance. Adding and subtracting could be quickly achieved, but then it was very expensive and not necessarily any faster than doing the job on paper. The appeal thus remained quite limited, not only because of the economic price to be paid (which was high), but also the cost of acquiring the necessary embodied skill to render them genuinely superior to existing customary practice.

What goes for the Thomas arithmometer, in this sense, goes also for every calculational technology, from counting on one's fingers, to the use of calculi on a counting board, or calculating with the abacus, the Pascaline, the Millionaire, logarithms, the MADAS, or any candidate for what was the most sophisticated mechanical calculator. Indeed even the HP-35 and HP-45 required a facility to do arithmetic backwards from the usual by its reliance on a method known as Reverse Polish. Thus for each technological development, whether Troncet or Omega, for it to find successful users each needed to be understood, and not only intellectually, but equally importantly, the potential users needed to be able to incorporate it into their embodied capacity, with each required gesture becoming so automatic as to require no or little thought.

The achievement of the late mechanical calculators (such as the MADAS and the Euklid-Mercedes) was that they greatly simplified what needed to be learned by their operators in order to achieve an efficient performance of all four arithmetic operations. But these calculators achieved this only at considerable economic cost. The Comptometer was really best for addition and (with practice) subtraction. With its key input it was fast, and its simple design was amenable to cheaper construction and mass-production. So it found a different and expanded market in the rapidly expanding commercial and government organisations of the C20. It has been said and is probably true that its inventors and promoters, Felt and Tarrant, were probably the first people in the world to become truly wealthy from the invention, production and sale of calculators.

In the above sense the history of calculation technology can be characterised not so much as the progress of mechanical invention, as it is sometimes presented, but as a more subtle evolving relationship between mind, body and material artefacts or put another way as an interaction between evolving technology, history, culture, mental skills, social capacities and aspirations. The search for a successful innovation was a strange mix of finding a place where these aspects converged to make the innovation seem useful, and at the same time not only economically but also culturally accessible.

47. S. Moreland, *A New and Most Useful Instrument for Addition and Subtraction of Pounds, Shillings and Pence* 1672, title page.

The requirement to change was not just set in the machine, but also in the humans who made up the society.

The final conquest of the electronic calculator occurred when literacy, design, familiarity, cost and perceived need coincided to sweep all else before it. Never had the market been more prevalent in the negotiation of daily life. Science and technology now dominated every corner of the developed world and was making rapid inroads elsewhere. Literacy was at its highest in the developed world with arithmetic education now a requirement for every child in the extended period of compulsory schooling. Money promised was supposed to free time. Technology was now widely accepted as being the answer to drudgery. Further, there was a growing social acceptance of life-long change, and an emerging concept of the desirability of 'life-long learning'. The new consumer calculators were increasingly cheap, required little knowledge to use, displaced the mental effort of recall of multiplication tables and mental and written arithmetic. Printers attached to them produced now comparatively permanent records. And the whole increasingly seamlessly fitted a world which would soon be interconnected through computers and telecommunications into an ever more pervasive communications web. Calculators had not only reached a desired end. They also had found their moment when that end was widely required.

One might ask if there is any lesson in this for the future. Clearly the technology of calculation is now passing not only the first but perhaps even towards the second vanishing point where it converges and merges with other electronic devices which themselves have become so much part of the habitude of daily life, especially in the developed world, that their presence is sinking into the invisibility of the routine environment of human experience. But in doing so much of habit had to be relearned, and in the consequence human thinking, as well as collective culture has transformed.

This is of course a history of one area of innovation. So it may have some relevance to other areas of innovation in the world in which we now live. It is perhaps appropriate to remember that humans have reached the point where their innovation is actually destabilising the physical world in which they live - a situation unimaginable for most of the time in which the developments discussed here have taken place. But it is the case that humans are now challenged with the requirement to achieve an unprecedented level of innovation if the planet is to remain stable. The story of innovation in calculators tells us that whilst the time may be ripe for us to accept a great deal of change, it will require re-learning to be comfortable with the many innovations that will be needed to achieve it. Getting there will require people to re-learn and reshape many attitudes, hidden assumptions and habitual ways of living. In seeking to make those changes, we might reflect on the many challenges overcome in the simpler long history of how humans have learned to calculate, and what that means for the learning and innovation that will now be required.

Appendix A

Objects in collection Calculant

The following objects form “collection Calculant”. They are described in context in the text.

Table A.1. Objects in collection Calculant

	First Devices
	~1894–1595 BC: Old Babylonian tag with cuneiform numeric and alphabetic characters
	Abacus (Chinese, recent construction)
	1480–1520: French Jeton, Late Medieval
	1577: Calculi Ordinum Belgii, “Port Salu”
	‘1617’: “Napier’s bones” (recent construction)
	‘1623’: Schickard’s “Calculating Clock” - reconstruction from surviving manuscript
	‘~1652’: Blaise Pascal’s “Pascaline” - replica based on surviving examples
	Logarithmic Tables, and Ready Reckoners
	1626: Dennis Henrion: Traicté de logarithmes

Continues...

1783: Tables Portatives de Logarithmes, by Gardiner improved and perfected in their disposition by M. Callet

1892: Ropp's Commercial Calculator Book, World's Fair Edition

The first "computer manual"?

1672: Morland (Samuel) A New, and most useful Instrument for Addition and Subtraction of Pounds, Shillings, Pence, and Farthings

Sectors and Dividers

1675–1715: Brass French Gunnery Sector by Michael Butterfield, Paris

~1830: Oxbone Architect's Sector by T. and H. Doublett, London

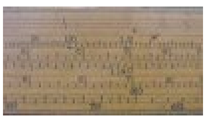
~1740–1810: 2 Pairs of Dividers - made in England

~1750–1800 Early print of drawings of Dividers and other Instruments by T. Jefferys

~1880 Set of Proportional Dividers and other Mathematical Drawing Instruments by Henri Morin, Paris

Conversion Devices

1790–1810: Instrument for conversion from Aunes to Metres by Gabrielle CHAIX, Paris



Gunter's Proportional Rule

Continues. . .

1626: Design of a Logarithmic Proportional Rule (following Gunter 1624)

1831–1843: Gunter rule for navigation by Belcher and Bros, New York, New York

Slide Rules

1727: Designs of 3 Logarithmic Rules 1626–1726 (as described by Jacob Leupold 1727)

1759–69: Edward Roberts Everard Pattern Gauger's Slide Rule

1821–84: Joseph Long, London, Alcohol Proof Index Slide Rule

1893–8: Tavernier-Gravet, Slide Rule, cursor inscribed as presented in 1898

~1928: Keuffel & Esser 4088–3 serial 348119 Slide Rule

~1948: Lawrence 10-G Cutting Speed Calculator

1967–73: Faber Castell 2/83N Novo Duplex slide rule

Cylindrical Slide Rules

1911: Thacher's Calculating Instrument, No. 4012 by Keuffel & Esser

1926: Professor Fuller's Fuller Cylindrical Slide Rule Model 2 No. 5706

~1960: Otis King Rotary Slide Rule, Type L C, serial T6205, scales 429 and 430

Circular Slide Rules

1847: Palmer's Computing Scale and Fuller's Time Telegraph, issue 4

Continues...

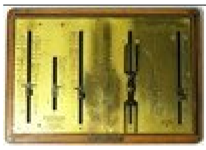
~1881: Charpentier Calculimètre circular slide rule

~1935: Supremathic Circular slide rule

1948: Fowler Long Scale Jubilee Magnum extra long scale calculator Fowler's "Jubilee Magnum" Circular Slide Rule

1960s: KL-1 Russian Mechanical Circular Slide Rule and case

Table A.2. Objects in collection Calculant



Proportional Nomographic Calculators

~1924: Bloch Schnellkalkulator

~1947 Der Zeitermittler

Stylus or Slider operated Adders

1869 model Web Patent Adder and Talley Board serial B2040

1890–1900: A. M. Stevenson Adder

1905–1911: Locke Adder

1910–1920s Comptator, 9 column

~1910 Adall Concentric-disk Adding Machine

~1915: Golden Gem Automatic Adding Machine, serial 73158

1920: Addiator Troncet adder

~1920: Addo mod 2 adder, serial 2797

1922: Scribola 10 column printing Adding Machine

1937: Add-O-Matic Adding Machine

~1946: Lightning Portable Adding Machine

1968: SEE Demonstration Adding Machine

Continues...

Rotating Knob Operated Adders

Keyboard Adding Machines

1884: Spalding Adding Machine, serial 494

1891: The Centigraph Adding Machine, serial 1523

1903: Adix Adding Machine. Original model

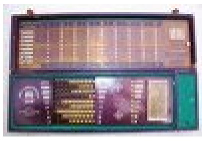
Arithmometers

1884: Thomas de Colmar Arithmometer Model T1878 B, serial 2083

1909: Ludwig Spitz & Co., G.m.b.H., Berlin-Tempelhof TIM (Time is Money) Arithmometer

~1913: Bunzel-Denton Arithmometer Prototype with crank moved to front

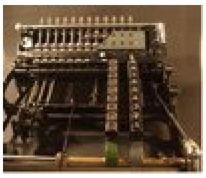
1917: Madas IX Maxima calculator, serial 5532



True Multiplying Calculators

1904–1905: Bamberger's Omega Calculating Machine

1912: Millionaire Calculating Machine serial 2015 (10×10×20)



Proportional Rack Calculators

1923: Mercedes-Euklid Model 29 Demonstration calculator

Continues...

1923: Mercedes-Euklid Model 29 calculator

Pinwheel Calculators

~1896: Ohdner Brunsviga Schuster Calculator serial 3406

1945: Facit Model S Calculator serial 210652

~1951: Original Ohdner Model 39, Calculator serial 39–288965

1957: Walther Demonstration WSR160 Pinwheel Calculator

~1957: Walther WSR160 Pinwheel Calculator

Comptometers

Sept 1896: Felt and Tarrant Comptometer: Earliest - wood-cased "Woodie" model, serial 2491

~ 1907: Comptometer oil bottle, marked with patent 23 April 1905; 6 April 1907

~ 1955 Bell Punch Sumlock Demonstration Comptometer

1950s: Bell Sumlock Comptometer Model 909/S/117.878 (made in Great Britain by Bell Punch Company Ltd)

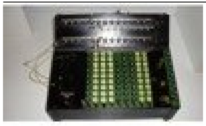
Curta Calculators

~July 1948: CURTA Type 1 (pin sliders) earliest model calculator serial 5424

1967: CURTA Type I calculator serial 76436, near mint, with original case, cardboard box and instructions

1963: CURTA Type II calculator with Leather Case, serial 554765

Continues...



Motorised Calculating Machines

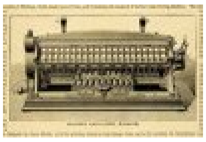
~1929 “Herzstark” electric Calculating Machine serial 6549, badged by Herzstark, Vienna; essentially Badenia Model TE 13 Duplex

1950s-1960s: MADAS Model 20BTG serial 94046 electric calculator with true automatic division

Electronic Calculators

July1972: Hewlett Packard HP 35 Calculator serial 1230A 79429, 1972 (second version)

December 1973: Hewlett Packard HP 45 Calculator serial 1350A 36719, 1973



Ephemerae

1759: Algebre et Arithmetique - Machine de Arithmetique de Pascal”, Denis Diderot and Jean le Rond d’Alembert, Encyclopédie ou Dictionnaire raisonné des sciences, des arts et des métiers, First folio edition, 1759, Volume 22, Plate 2. (Hand coloured).

1797: Napier’s Rods in “Inland Navigation”, Encyclopaedia Britannica, 3rd Edition, 1797, plate CCCXLIV, Andrew Bell copperplate. Original print article.

1855: “Messrs. Scheutz’s New Calculating Machine”, The Illustrated London News, 30 June 1855, p. 661. Original print article.

1890: Leon Bollée Calculating Machine, “A New Calculating Machine of very General Applicability”, The Manufacturer and Builder, July 1890, p. 156. Original print article.

Continues. . .

1873–1928: Booklet on "How to Become a Lightning Calculator", No. 2
1901: "Rechenmachinen ["Calculating Machines" trs.] Bibliogr. Institut in Leipzig, Zum Art "Rechenmaschinen" (Bld. 21). Original wood engraved plate, pp. 1–2. showing engravings of calculating machines by Pascal, Leibniz, Steiger and Egli (Millionaire), Burroughs, Burkhard, Brunsviga, and J. H. Müller.
1913: Advertisement by Hertztark Co. for "Austria" Calculating Machine
1921: Felt and Tarrant Comptometer Manual
1937: Advertisement for Monroe Calculators featuring Marching numbers
1973: "HP Measurement/Computation:changing things for the better", Scientific American, July 1973, pp. 6–7. Original print article. Two full page advertisement for the HP-35, HP-45 and HP-46 scientific calculators.
1990: "A Calculator Chronicle: 300 Years of Counting and Reckoning Tools", 1 January 1990.

Appendix B

Interview with an author

Interview with Jim Falk (by his younger self)

I: So, you said you would give me the explanation of why you collect calculators.

J: I don't think I promised 'the explanation'.

I: I think you did! But we can let that pass. Why have you done this?

J: It was part of my job. Oh no, I am retired (sort of). Well, I guess it must just be fun.

I: Fun??? Calculators!! Come off it!

J: Well I could give you a story. Would that help?

I: Maybe.

J: When I was a kid - started it I think when I was 13 - I did a project for a thing called the "Science Talent Search" which in 1961 could get you a nice lot of money (£25), that was Australia's currency then, if you won. (I did - I bought a movie camera with it - It had a clockwork motor!) I chose to do the first ever project on psychology. (I really wanted to do it on hypnotism but knew I would never get away with it.) Anyway, after subjecting 10 classmates to unending experiments - I wrote a 443 page thesis on "Pilot Experiments in Memory". It had about 100 pages of statistics at the back, and someone lent me a Facit pinwheel calculator to help me add up, subtract, and square columns and columns of numbers. I loved that gadget! So one fateful day more recently I put "Facit calculator" into `ebay`. Found one, and bought it.[W] And so it began . . . .

I: Oh, so you were just a young nerd were you?

J: No, yes, oh maybe. I came bottom of the class in mental arithmetic, and failed geometry, if that helps?

I: Not really - but maybe explains why you liked the calculating machine - couldn't do without it?

J: So you think that explains it?

I: Might.

J: No, really, that's just one explanation. Why do you guys always want "the" story? Things are often more complicated.

I: So there's more?

J: Sure, how much time have you got?

I: Not much, I do have to do lunch real soon. Could you just give me the drift?

J: OK, here's another stab at it... Once upon a time. No I'd better take this seriously. The point is I graduated from Monash with an Honours degree in Science and went on to do a PhD in theoretical quantum physics. Completed it in 1974.

J: Oh and on the way I was a trainee programmer at Caterpillar in my second year at university in 1966, later was one half of the first computer help desk at Monash, and when doing my PhD used to operate the main frame computers at night for pay (and a chance to run my programmes all night.) And over that time I saw the tools of calculation go from mechanical calculators, log tables, and slide rules, to motorised machines, to the electronic wonder of the Hewlett Packard pocket scientific computers in 1972, which could do the lot. After that, all that had come before was gaslight! Now of course you can do it all in your iphone.

I: So you are a nerd. I knew it! Why not come clean about that in the first place? I mean who do you think you are fooling? It's not entirely bad to be a propeller head, supposedly.

J: But that wasn't all. I mean I had been heavily involved in student politics. I was President of the Student's Representative Council in my second and third years, and I became a strong anti-Vietnam war activist. Even went to Vietnam in the middle of the war (Jan 1969) and got accreditation as a journalist.

I: Well thats a clue! I mean you Boomers still just can't seem to be able to get over the 60s, can you? Protests, long hair, and drugs. I've read about it. I guess way back then it helped pull the chicks? Anyway, I don't see how reminiscing about your youth as a "radical" gets us much closer to this thing about calculators (and look at you now), so maybe...

J: Hang on. There's a link.

I: Look, it is lunch time... Don't take this personally, but I have to interview someone who seriously matters after this... There is some meeting about a big potential contract between the University and Google...

J: I'll try to be quick.

I: OK - could you please get on with it.

J: Well to cut it short.

I: Yes please.

J: When I finished my PhD I wasn't entirely happy the way science had been used in Vietnam, and the environmental issues were beginning to surface, so I went and worked in environmental type activities - including for the Australian Conservation Foundation, and working for the Shadow Minister for Consumer Affairs. But in the end I had to decide which way I was going to jump - science or the other way. Crunch came in 1980 when I was offered jobs in Theoretical Quantum Chemistry at Monash, and in History and Philosophy of Science in the Arts Faculty at the University of Wollongong. I took the latter, and that, as they say, has made all the difference.

I: OK so you chose the soft stuff over the hard stuff did you?

J: Um. . .

J: Anyway at Wollongong I was employed to lecture on the politics and such like of modern science and technology. There I was surrounded by 'real' historians and philosophers of science (well a couple) who knew all sorts of other interesting things about how to think about that. So I learned a bit about that too. By 1989 I was Head of what was now the Department of Science and Technology Studies (STS). So I was in an environment where people professionally look at the history of technology and science, and how innovation happens. If you really want to know about any of this stuff I did and do you could look *here*[1] or for the more recent stuff here.[(w)]

I: No, thanks. We've already had quite enough of that.

I: So to summarise you went from nerd to a sort of mixed up soft geek who has got into a bit of steam punk. I get it. But it doesn't seem to have helped you. It just seems to have left you all confused about these calculators and why you collect them.

J: Well, that seems a bit blunt.

I: Anything else?

J: Only that after that from 1996 I became a 'senior' university executive ('Deputy Vice-Chancellor' - that sort of stuff) at a couple of universities (where amongst other things I had responsibility for their IT developments). But you know research is much more interesting! So in 2004 I went back to found and run a new research outfit at the University of Melbourne - the Australian Centre for Science, Innovation and Society. So you can tell by the name that it was about technological innovation amongst other things. Kicked the habit at the beginning of 2011. So I no longer have to run round with a collection plate for funding the Institute. What a relief! I'm still a Professorial

1. `www.genealogy.metastudies.net/PS01/PS01_001.HTM`

Fellow *here*[2] with this office, but not many responsibilities, and also I'm free to do other things.

I: So now you are out in the pasture, you have all the time in the world, instead of having to do serious work to do things, however pointless (if you'll excuse me saying so), like collect the calculators and do this website?

J: Sort of. I still am involved in research on climate change, I'm a "Visiting Professor" at the *United Nations University*[3] and an Emeritus Professor at the *University of Wollongong.*[4] I still write on *various issues*[5] and am an Affiliate Researcher with the *Melbourne Sustainable Society Institute.*[6] My latest book (with Joseph Camilleri) is *"Worlds in Transition: Evolving Governance Across a Stressed Planet".*[7]

I: Yes, well, I suppose thats nice, if you must be, well sorry, an egg head. But why retire if you are going to do that? You don't seem to have thought it through very well. Anyway, can't really see what it has to do with calculators.

J: Well one of my most cited articles is from 1995 on *"The Meaning of the Web"*[8] and the new book has a long chapter I wrote on the evolving governance of information.

I: Can't say that says much about calculators! That's drawing a long bow isn't it?

J: OK - well *my father*,[9] who was a philosopher, used to collect old art works - from ancient Egypt and China. They were amazing - ancient elegant survivors of an age otherwise beyond our reach. They seemed so magic to me as a kid. So I collect calculators.

I: Well that seems like two completely different things entirely! I don't see the connection.

J: OK, maybe this will do - the calculators have helped me understand the history of calculation, and the history of calculation has helped me decide what I wanted to actually have in my hands.

I: Yes - but isn't that just an excuse? Do you really need to HAVE these things to just write a history?

J: Look, I like calculators, especially the old ones, OK?

I: Whatever. I have to run to lunch. Some of us still have real jobs to do. If I have any more questions I'll text you. OK?

J: Well have you got what you want?

2. www.land-environment.unimelb.edu.au
3. www.ias.unu.edu/sub_page.aspx?catID=78&ddlID=81
4. uow.edu.au
5. metastudies.net
6. www.sustainable.unimelb.edu.au
7. worlds-in-transition.com
8. groups.csail.mit.edu/mac/classes/6.805/articles/falk-meaning-of-the-web.html
9. www.genealogy.metastudies.net/ZDocs/Stories/Falk02.html

I: Look, its not what I expected. I thought you would tell me why you have collected these calculators. But it seems like you don't know. It's more like the calculators have collected you.

J: Why do you think mathematicians do mathematics?

I: Beats me. Why do normal people watch sport?

J: OK well why do they buy sports cars?

I: Well thats obvious. Sports cars are right at the edge of engineering, design and style - really neat. I've got one of the old Austin Healey Sprites. Really fun! Would loosen you up a bit, maybe. What has that got to do with collecting calculators? One of the reasons you seem so weird is you are forever trying to change the subject.

J: Well we have big problems like *climate change*[10] which need a whole lot of innovation to solve. So maybe we can learn something about how collectively, we as humans, got faster and better at solving the calculation problem? Maybe solving one depends on solving the other? I mean isn't calculation a crucial tool in our *evolving capacity for governance*?[11]

I: You've lost me. Is that the sort of stuff you feed your students? Like footnotes are going to change the world?

J: So do you still want to write this article?

I: It was supposed to be a "human interest" story, but to be frank I'm not sure how either of those words apply to you. Maybe I can do something with this. But don't hold your breath. Have to see how the Editor responds. I'll text you if I do write anything. No need to contact me. Bye.

		
Jim 2012	"The adding machine" 1961	Jim ("James") 1961

10. books.google.com.au/books?id=8xMJAQAAMAAJ&q=Falk+the+greenhouse+challenge&dq=Falk+the+greenhouse+challenge&hl=en&ei=1evaTsOjCMbEmQX-04zICw&sa=X&oi=book_result&ct=result&resnum=1&ved=0CDQQ6AEwAA
11. books.google.com.au/books?id=JfQFMo4j3UQC&printsec=frontcover&redir_esc=y#v=onepage&q&f=false

Appendix C

Bibliography of electronic sources

This is a collection of electronic resources (including selected links to other relevant websites) which form part of the background to the collected objects and associated analysis.

C.1 Early books

- John Napier, http://metastudies.net/pmwiki/uploads/Books/Rabdologiae_seu_numerationis_per_virgula.pdf

  Rabdologiae seu numerationis per vigula, 1628 (first published 1617).

- Blaise Pascal, http://www.bibnum.education.fr/calculinformatique/calcul/la-pascaline-la-Â«%C2%A0machine-qui-relève-du-défaut-de-la-mémoire%C2%A0Â»

  "Lettre dédicatoire à Monseigneur le Chancelier sur le sujet de la machine nouvellement inventée par le Sieur B.P. pour faire toutes sortes d'opérations d'arithmétique, par un mouvement reglé, sans plume ny jettons avec un advis necessaire à ceux qui auront curiosité de voir ladite machine, & de s'en servir", 1645.

- Jacob Leupold, http://metastudies.net/pmwiki/uploads/Books/TheatrumArithmeticoGeometricum.pdf

  Theatrum Arithmetico-Geometricum das ist: Schau-Platz der Rechen- und Mess-Kunst. . . . Christoph Zunkel, Leipzig, 1774 (first edition 1727). The best illustrated book on calculation and measurement published during the eighteenth century.

- http://ebooks.library.cornell.edu/m/math/browse/title/a.php

  Cornell University Library Historical Math Monographs. A wonderful collection of early mathematics books generally able to be read on line.

- http://www.bibnum.education.fr/recherche/resultat/liste/

  Bibnum Classic scientific articles from before 1940 relevant to calculation, mathematics, and information science. Includes original documents from Pascale and Leibniz.

C.2 Key websites

C.2.1 Survey websites

- http://www.rechnerlexikon.de/en/artikel/Main_Page

  Rechnerlexicon The leading and most comprehensive survey site for calculator specialists and collectors with information about almost every known mechanical calculator and a database of more than 23750 patents.

- http://www.rechenmaschinen-illustrated.com/

  Rechenmaschinen-Illustrated: The most comprehensive listing of details about early calculating machines on the web. Based around the classic study Ernst Martin, *Die Rechenmaschinen*, 1925, and translated into English as http://www.rechenmaschinen-illustrated.com/

  Ernst Martin, "The Calculating Machines", MIT Press and The Charles Babbage Institute, USA, "History of Computing" series, 1992.

- http://www.histoire-informatique.org/idx/

  Histoire de l'Informatique Wide-ranging gallery of calculators and associated information. [in French]

- http://www.rechenkasten.de/BueromaschinenLexikon/index.html

  Büromaschinen Lexikon Published by Goeller-Verlag the annual office machines dictionary provided a comprehensive illustrated overview of common office equipment for the respective years. This site provides collectors with the information on calculators from that website.[in German]

- http://home.vicnet.net.au/~wolff/calculators/

  John Wolff's Web Museum Calculating Machines A wonderfully detailed site by an expert collector with great mechanical insight into the workings of the extensive set of calculators listed.

- http://w1.131.telia.com/~u13101111/typewriters.html

  Swedish Typewriters Site - listing of antique office calculators From the home page select the "The Collection" in the navigation bar on the left, and then scroll down to the RäKNEMASKINER / CALCULATORS section.

- http://www.ph-ludwigsburg.de/fileadmin/subsites/2e-imix-t-01/user_files/mmm/mmm_online/

Mathematisches Maschinenemuseum Good historical summaries and a nice set of videos of machines in action.

- `http://www.mechrech.info`

  Beiträge zur Geschichte des mechanischen Rechnens: Contributions to the History of Mechanical Calculation Good list of publications, books and historical articles dealing in particular with the history and internal workings of calculating machines.

- `http://ed-thelen.org/comp-hist/on-line-docs.html`

  Online documents A broad listing of useful links to documents covering the history of calculators.

- `http://calcollect.free.fr/machines/macha.htm`

  Michel Bardel's list Tentative list of all existing mechanical machines since 1820, together with FAQs and serial numbers. An invaluable reference source.

- `http://www.thocp.net/timeline/1620.htm`

  History of Computer Antiquity

- `http://bluemich.net/rechner/`

  Mechanische Rechenhilfsmittel Mechanical Calculating Devices by Wolf G. Blümich (in German.)

- `http://www.madeasy.de/2/mathe.htm`

  Mathematics site by Reiner Hoffmann in German.

- `http://www.opticaliamuseum.com/drawing/`

  Opticalia Museum - museum menu is working, but the catalogue menu was still under construction in May 2012.

- `http://www.mipaginapersonal.movistar.es/web3/calculating/LLIBRE%20JPG/INDEX%20SOLO.htm#alfa`

  Índice de las Máquinas de Calcular a comprehensive survey site by Josep Balsach Peig, in Spanish.

- `http://calmeca.free.fr/calculmecanique_php/index_calmeca.php?lang=eng`

  François Babillot's calculator website

- `https://www.zerfowski.com/rechengeraete.php`

Mechanische Rechenhilfsmittel und -maschinen by Detlef Zerfowski including an extensive list of patents and also calculator sites

C.2.2 Manuals and Instructions

- http://www.rechenwerkzeug.de/default.htm

  Calculator Manuals from Rechenwerkzeug.de

- http://www.nicolamarras.it/calcolatoria/downloads_en.html

  Calcolatoria downloads

- http://www.mathsinstruments.me.uk/page62.html

  Mathematical Instruments Catalogues and Brochures

- http://www.prehistoriadelainformatica.com/manuales -instructions/

  Prehistoria de la Informatica Office Collectibles

- http://users.lewiston.com/ejorgens/office/miscellaneous /miscellaneous.html

  Miscellaneous Manuals Pay site, but if you can't get these elsewhere...

- http://officemachinemanuals.com/catalog/miscellaneous.htm

  Office Machines America Pay site.

- http://www.mechanicalculator.com/manuals/

  Collection of Kees Nagtegaal useful set of manuals for slide rules and the more common calculators.

- http://http://www.mccoys-kecatalogs.com/KEManuals/manuals.htm

  Keuffel & Esser Slide Rule Manuals and Catalogues site by Clark McCoy.

C.2.3 Specialist websites

- http://www.arithmometre.org

  Valéry Monnier's extraordinary and expert website on the Thomas de Colmar Arithmometer

- http://www.ami19.org/Arithmetic Machines

Arithmetical Machines & Instruments / 19th century Valéry Monnier's other set of images and details of some of the most rare and precious calculating machines of the C19 (in French and English).

- `http://mortati.com/glusker/elecmech/refslinks.htm`

  Electromechanical calculating machines from the 1960's includes extensive lists of other relevant links and books.

- `http://ajmdeman.awardspace.info/`

  Original documents on the History of Calculators

- `http://www.yvesserra.fr/`

  Le cabinet de curiosités d'Yves Serra A nice presentation of information on early calculators (with particular attention to those of Pascale and Leibnitz).

- `http://www2.cruzio.com/~vagabond/ComptHome.html`

  Comptometer Website Very authoritative website on the Felt & Tarrant Comptometer.

- `http://www.vcalc.net/`

  The Calculator Reference Comprehensive coverage of `http://www.vcalc.net/cu.htm`

  Curta, `http://www.vcalc.net/hp.htm`

  Hewlett Packard, and `http://www.vcalc.net/ti.htm`

  Texas Instrument calculators. Includes links to other sites and manuals.

- `http://home.comcast.net/~wtodhner/calcs.html`

  William Ohdner's site commemorating the Ohdner calculator

- `http://www.xnumber.com/xnumber/cmisc_facit_page.htm`

  Facit Page by James Redin including a comprehensive list of models of the Facit calculators.

- `http://www.xnumber.com/xnumber/facit_history.htm`

  History of the Facit Calculator by Christofer Nöring

- `http://www.taswegian.com/MOSCOW/soviet.html`

  Museum of Soviet Calculators

- http://rk86.com/frolov/calcolle.htm

  Soviet Calculators Collection

- http://officemachinemanuals.com/

  Office Machines Americana including a useful set of manuals.

- http://www.calculators.de/

  Museum of Pocket Calculating Devices mainly focussing on electronic pocket calculators.

- https://plus.google.com/photos/103618537816338035609/albums ?banner=pwa

  John Huey's Picasawebalbum with illustrations of a wide range of Addiator Troncet calculators.

C.2.4 Gunter Scales & Sectors

- http://www.rainerstumpe.de/HTML/erl01.html

  Gunter Works All about Gunter Scales and Sectors and how to use them (mostly in German but also in part in English)

C.2.5 Slide Rules

- http://sliderulemuseum.com/

  International Slide Rule Museum

- http://sliderulemuseum.com/HSRC_Menu.htm

  Herman van Herwijnen's Slide Rule Catalogue "LITE"

- http://www.hh.schule.de/metalltechnik-didaktik/museum /rechenschieber/scheiben/rs9.htm

  Rechenscheiber im Online-Museum

- http://www.sliderule.ca/

  Eric's Slide Rule Site

- http://sliderule.ozmanor.com/index.html

  Greg's slide rules

- http://www.sliderules.info

Robert Manley's Slide Rule Site

- `http://bluemich.net/rechner/rma21.htm#14`

  Anhang 2.1: Bedienungs- und Serviceanleitungen für Tabellen, Rechenschieber, -scheiben, -walzen und -skalen Instructions for many slide rules.

- `http://oughtred.org/`

  The Oughtred Society

- `http://sliderules.lovett.com/cookiedev/extendedlitsearch.html`

  Scanned back issues of the Oughtred Society Journal and other publications An essential reference site for enthusiasts. Access to the last five years publications requires a password and username to be provided by the Society.

- `http://www.rekeninstrumenten.nl/index.html`

  Dutch Circle for Historical Calculating Instruments

- `http://www.hpmuseum.org/sliderul.htm`

  Museum of HP Calculators History of Slide Rules

- `http://www.xnumber.com/xnumber/hp.htm`

  The Death of the Slide Rule by James Redin

- `http://www.sliderule.it/`

  Collection of Giovanni Breda

- `http://sliderulemuseum.com/SR_Links.htm`

  International Slide Rule Museum A survey site focussing primarily on slide rules with a wealth of links and other material including other related sites, and manuals and related material. See also its helpful and comprehensive `http://sliderulemuseum.com/SR_Links.htm`

  links page.

- `http://www.svpal.org/~dickel/OK/OtisKing.html`

  Otis King's Patent Calculator Specialist site focussing on the Otis King cylindrical sliderule.

- `http://linealis.org/`

linealis.org "Un site consacré aux instruments du calcul manuel tels que les règles et cercles à calcul, les curseurs, abaques, machines à calculer tenant dans la main, planimètres et intégrateurs."

- `http://www.sphere.bc.ca/test/2archives.html`

  The Slide Rule Universe by Sphere research corporation.

C.2.6 Measuring & Drawing Instruments

- `http://collectingme.com/measuring/`

  CollectingMe.com Some nice examples of various antique measuring instruments.

- `http://www.petergh.f2s.com/instruments2.html`

  Peter Giringhelli's collection of measuring and drawing instruments, and rules

- `http://geometricum.jimdo.com/collection/scientific-instruments/`

  Geometricum Markus Geissbuehler presents a systematic presentation of scientific instruments, in particular astronomical, drawing instruments and surveying instruments from Roman to C19

- `http://www.mathsinstruments.me.uk/page6.html`

  Mathematical Instruments: A Private Collection

C.2.7 Museums

- `http://www.arithmeum.uni-bonn.de/`

  The Arithmeum, Bonn A specialist museum focussed on the history of calculation, housing many rare and precious antique calculators.

- `http://www.sciencemuseum.org.uk/onlinestuff/museum_objects/mathematics.aspx?page=1`

  Science Museum (UK)

- `http://en.hnf.de/default.asp`

  The Heinz Nixdorf MuseumsForum Claims to be the world's biggest computer museum, situated in Paderborn, Germany.

- `http://www.arithmometre.org/Musees/PageMusees.html`

List of museums at www.arithmometer.org

- http://www.landesmuseum-stuttgart.de/

  Stuttgart Landesmuseum

- http://www.ph-ludwigsburg.de/fileadmin/subsites/2e-imix-t-01/user_files/mmm/mmm_online/

  Mathematische Maschinenmuseum. Collection of the Hochschule in Ludwigsburg (Germany)

- http://www.uni-greifswald.de/~wwwmathe/RTS/node2.html

  University of Greifswald An extensive mechanical calculator collection.

- http://http://www.radiomuseum.org/museum/d/computer-museum-der-rwth-aachen/.html

  Computer Museum Aachen Primarily electronic computers.

- http://www.compustory.com/

  The Museum of Modern Human Progress (The American Computer Museum)

- http://www.computermuseum.nl/collecti.html

  Stichting Computermuseum (Netherlands)

- http://www.computerhistory.org

  The Computer History Museum (Mountain View, California, USA)

- http://interstices.info/jcms/c_15272/machines-a-calculer

  Insterstices Machines à calculer l'association pour le musée international du calcul, de l'informatique et de l'automatique de Valbonne Sophia Antipolis (Amisa) présente quelques pièces de son patrimoine.

C.2.8 Other Useful Sites

- http://www.xnumber.com/xnumber/

  History of Mechanical Calculators Part of James Redin's Xnumber World of Personal Caclulators site. Some useful links to documents (also listed in other sites.)

- http://www.vintagecalculators.com/index.html

Vintage Calculators Website A significant number of photos of the more widely collected old mechanical calculators, together with a survey of electrical and electronic machines.

- `http://www.crisvandevel.de/links.htm`

  Recommended links by Chris from Calclist

- `http://www.rechenwerkzeug.de/`

  rechenwerkzeug Reinhard Atzbach's site with a very nice range of antique calculators generally in very nice condition, and an extensive listing of `http://www.rechenwerkzeug.de/literat.htm`

  relevant literature (in German).

- `http://www.computissimo.ch/v-fran/calcul.htm`

  Calculatrice

- `http://www.earlyofficemuseum.com/calculating_machines.htm`

  The Early Office Museum

- `http://www.boelters.de/Rechenmaschinen/index.html`

  Aus der Rechenmaschinen-Werkstatt von D. Bölter

- `http://retrocalculators.com/default.aspx`

  Retrocalculators.com

- `http://www.rechenwerkzeug.de/default.htm`

  Mein Rechner tut's auch ohne Strom...

- `http://www.rechenhilfsmittel.de/`

  Jan Meyer's Geschichte der Rechenhilfsmittel

- `http://www.schneemann.de/index.htm`

  Schneemanns web page

- `http://www.w-hasselo.nl/mechn/index.php`

  Wim Hasselo's comprehensive website

- `http://www.crisvandevel.de/`

  Cris Vandevel's Antique Mechanical Four-species Calculators

- `http://calmeca.free.fr/calculmecanique_php/recherche_BDD/appel_resu_trouve_fichier.php?lang=eng`

  François Babillot's Database of Antique Calculators

- `http://www.hpmuseum.org/prehp.htm`

  General calculator list from the "Museum of HP Calculators"

- `http://www.yesterdaysoffice.com/`

  Yesterday's Office includes a forum for collectors of office antiques.

- `http://en.wikicollecting.org/calculators`

  WikiCollecting article on collecting calculators

- `http://www.gbreda.it/mechcalc/link.php`

  Giovanni Breda's list of calculator collector and calculator & slide rule websites

- `http://pactu.com/`

  Philip A Cannon II's site with a great deal of information cataloguing makers of instruments.

- `http://www.mwtca.org/tool-dealers.html`

  Mid-West Tool Collectors Association List of US antique tool dealers (not particularly calculators, but some relevant tools are accessible).

- `http://public.beuth-hochschule.de/~hamann/`

  Prof. Dr.-Ing. Christian-M. Hamann's calculator site

- `http://www.mortati.com/glusker/elecmech/index.htm`

  Mark Glusker's calculator site

C.2.9 Associations of Calculator Collectors, Collectors Lists, and Collector Mail Lists

- `http://www.lsoft.com/scripts/wl.exe?SL1=CALCLIST-L&H=LISTSERV.TECHNION.AC.IL`

  CALCLIST a "must" subscription for the serious calculator collector.

- `http://groups.google.com/group/oldcalculatorforum`

Old Calculator Forum e-mail list website mainly but not exclusively focussing on electric and early electronic calculators.

- `http://groups.yahoo.com/group/oldcalcs/`

  Old Electronic Calculators Yahoo group primarily for old electronic calculators.

- `http://tech.groups.yahoo.com/group/sliderule/messages`

  Yahoo slide-rule group. It is very active.

- `http://calcollect.free.fr/`

  ANCMECA - machine à écrire mécanique et à la machine à calculer mécanique in multiple languages.

- `http://ifhbev.de/index.php?id=5`

  IFHB Internationales Forum Historische Bürowelt e.V. with also an extensive set of links to other relevant sites.

- `http://www.webring.org/hub?ring=calculator`

  Webring of Calculator Collecting simply a web-ring for those interested.

- `http://www.datavis.ca/milestones/index.php?group=1600s`

  Milestones in the History of Thematic Cartography, Statistical Graphics, and Data Visualization a nice timeline with links.

- `http://www.xnumber.com/xnumber/collectors_mech.htm`

  Mechanical Calculator Collectors in the Net listing by James Redin.

C.2.10 Personal Collections

- `http://www.xnumber.com/xnumber/collectors_mech.htm`

  List of famous calculator collectors

- `http://www.xnumber.com/xnumber/`

  Another list of collectors

- `http://www.calculators.szrek.com/`

  Collection of Walter Szrek

- `http://www.jmgoldman.com/`

Collection of Jay M. Goldman

- http://www.nzeldes.com/HOC/HOC_Core.htm

  Nathan's Possibly Interesting History of Computing Website

- http://www.hpricecpa.com/typewriterinfo9.html#acam

  Collection of Herman Price

- http://www.mechanicalculator.com/

  Collection of Kees Nagtegaal

- http://machineacalculer.free.fr/

  Collection of Christophe Mery

- http://www.calculi.nl/

  Collection of Nico Baaijens

- http://www.gateman.com/museum/c3.html

  Topeka Computing Museum - "Ed's 948 Collection"

- http://www.peterkernwein.de/

  Collection of Peter Kernwein

- http://www.oldbits.com/collection.htm

  "Old Bits" collection of Frank Salomon

- http://www.thimet.de/CalcCollection/Contents.html

  Calculator Collection of Anthon (Tony) Thimet

- http://users.skynet.be/Fredscalculators/main.htm

  Fred's Mechanical Calculators Calculator collection of Fred Haegen

- http://www.prehistoriadelainformatica.com/

  Prehistoriadelainformatica High quality collection of office machines by Eduardo Guillem, in Spanish. Includes a fine selection of http://www.prehistoriadelainformatica.com/calculadoras-calculators/

  antique mechanical calculators and a number of http://www.prehistoriadelainformatica.com/manuales-instructions/

  manuals.

- `http://www.tcocd.de/index.shtml`

  Collection of Andreas Tabak

- `http://www.ulisse.bs.it/museo/index.htm`

  Museo didattico del Computer in Itallian

- `http://www.telmachines.nl`

  Telmachineverzameling Bob de Groot in Dutch

- `http://www.vierkantvoorwiskunde.nl/sikkepit/Rekenapparaten.html#terugvankleine`

  sikkepit Collection of Jantine Bloemhof (in Dutch) primarily aimed at demonstrating to school children different ways of doing calculations before the advent of electronic calculators.

- `http://things-that-count.com`

  Things that Count - well you must know about that, or you wouldn't be here...

C.2.11 Online stores

Some useful stores of antique scientific instruments. (Calculant has no financial connection with any of these stores and cannot vouch for anything advertised. Caveat Emptor applies!)

- `http://www.fleaglass.com/`

  Fleaglass - a store front for a range of private sellers and dealers for various categories of antique scientific instruments including those for calculation

- `http://www.gemmary.com/`

  The Gemmary Antique Scientific Instruments and Old & Rare Books - both a dealer and store front for sales on consignment. Pricing tends to conform to the high quality of both the objects and the great care and expertise with which they are described, questions answered, and orders executed. The proprietor, Dr RC Blankenhorn is a well known expert in this field.

- `http://ebay.com`

ebay US, `http://ebay.com.au`

ebay Australia, `http://ebay.de`

ebay Germany, `http://ebay.at`

ebay Austria, `http://ebay.fr`

ebay France, `http://www.ebay.co.uk/`

ebay UK, `http://www.ebay.it/`

ebay Italy and in general ebay around the world is probably the prime location for purchase of the more common (and some less common) antique calculation devices and books. Prices are very variable and if you are lucky ridiculously cheap (or conversely absurdly expensive). You are absolutely on your own in judging the merit of the items on offer. Dates and descriptions are often on a scale of misleading to fanciful. Things advertised as "rare" are more often than not not.

- `http://www.gilai.com/topcat_14/Scientific-Instruments`

  Gilai Collectibles.

- `http://www.etesseract.com/`

  Teseract Early Scientific Instruments - a pricey but high quality inventory of antique scientific instruments including a notable collection of rules, sectors, and surveying and measuring instruments.

- `http://www.patented-antiques.com/Backpages/survey/Drafting.htm`

  Meeker's Mechanical Nature Antiques - a cheap but generally not very old set of rules and drawing instruments.

- `http://www.thebestthings.com/measurin.htm`

  The Best of Things - primarily of interest for vintage measuring tools, primarily but not only from America.

- `http://www.sis.org.uk/dealers`

  Scientific Instrument Society - UK listing of dealers in scientific instruments, notably including mathematical instruments.

- `http://www.asiuk.net/`

  Antique Scientific Instruments U.K..

- `http://www.newbegin.com/html/scientific.html`

Newbegin Antiques.

- `http://www.antiques-sci-tech.com/wcat26.html`

  Antiques of Science and Technology A short list of calculators, prices on application.

- `http://www.breker.com/LiveAuctions/`

  Auction Team Breker (Köln (Godorf), Germany) run a scientific instruments speciality auction several times per year and occasionally also list items on ebay.

- `http://www.bonhams.com/`

  Bonhams (London) run occasional auctions of scientific instruments.

C.2.12 Reference

- `http://historydb.adlerplanetarium.org/signatures/`

  Webster Signature Database Provides an excellent starting point for searching for details of past makers of scientific instruments.